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Art of Problem Solving

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– level 2

1 Find the smallest 3-digit number that is the product of two 2-digit numbers , so that the seven digits of these three numbers are all different.

2 Gonçalo writes in a board four of the the following numbers $0, 1, 2, 3, 4$, he can repeat numbers. Nicolas can realize the following operation: change one number of the board, by the remainder(in the division by 5) of the product of others two numbers of the board. Nicolas wins if all the four numbers are equal, determine if Gonçalo can choose numbers such that Nicolas will never win.

3 In a triangle ABC with $AB = AC$, let M be the midpoint of CB and let D be a point in BC such that $\angle BAD = \frac{\angle BAC}{6}$. The perpendicular line to AD by C intersects AD in N where $DN = DM$. Find the angles of the triangle BAC.

4 At a dance there are 12 men, numbered 1 to 12, and 12 women numbered 1 to 12. Each man is assigned a "secret friend" among the 11 others. They all danced all the pieces. In the first piece each man danced with the woman who has the same number. From then on, each man danced the new piece with the woman who had danced the piece earlier with his secret friend. In the third piece the couples were:
https://cdn.artofproblemsolving.com/attachments/c/d/f5ea0931e5751739c1ba556f84ab5736f2d11
png
Find the number of each man's secret friend.

5 The enemy ship has landed on a 9×9 board that covers exactly 5 squares of the board, like this:
https://cdn.artofproblemsolving.com/attachments/2/4/ae5aa95f5bb5e113fd5e25931a2bf8eb872db
png
The ship is invisible. Each defensive missile covers exactly one square, and destroys the ship if it hits one of the 5 squares that it occupies. Determine the minimum number of missiles needed to destroy the enemy ship with certainty .

– level 1

1 On the blackboard were six figures: a circle, a triangle, a square, a trapezoid, a pentagon and a hexagon, painted in six colors: blue, white, red, yellow, green and brown. Each figure had only one color and all the figures were of different colors. The next day he wondered what color each figure was.
Paul replied: "The circle was red, the triangle was blue, the square was white, the trapezoid was

green, the pentagon was brown, and the hexagon was yellow.
Sofía answered: ”The circle was yellow, the triangle was green, the square was red, the trapezoid was blue, the pentagon was brown, and the hexagon was white.”
Pablo was wrong three times and Sofia twice, and it is known that the pentagon was brown.
Determine if it is possible to know with certainty what the color of each of the figures was.

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2 An integer is called *autodivi* if it is divisible by the two-digit number formed by its last two digits (tens and units). For example, 78013 is autodivi as it is divisible by 13, 8517 is autodivi since it is divisible by 17. Find 6 consecutive integers that are autodivi and that have the digits of the units, tens and hundreds other than 0.

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3 A segment AB of length 100 is divided into 100 little segments of length 1 by 99 intermediate points. Endpoint A is assigned 0 and endpoint B is assigned 1. Gustavo assigns each of the 99 intermediate points a 0 or a 1, at his choice, and then color each segment of length 1 blue or red, respecting the following rule:
The segments that have the same number at their ends are red, and the segments that have different numbers at their ends are blue. Determine if Gustavo can assign the 0's and 1's so as to get exactly 30 blue segments. And 35 blue segments? (In each case, if the answer is yes, show a distribution of 0's and 1's, and if the answer is no, explain why).

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4 There are two paper figures: an equilateral triangle and a rectangle. The height of rectangle is equal to the height of the triangle and the base of the rectangle is equal to the base of the triangle. Divide the triangle into three parts and the rectangle into two, using straight cuts, so that with the five pieces can be assembled, without gaps or overlays, a equilateral triangle. To assemble the figure, each part can be rotated and / or turned around.

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5 a) In each box of a 7×7 board one of the numbers is written: $1, 2, 3, 4, 5, 6$ or 7 of so that each number is written in seven different boxes. Is it possible that in no row and no column are consecutive numbers written?

b) In each box of a 5×5 board one of the numbers is written: $1, 2, 3, 4$ or 5 of so that each one is written in five different boxes. Is it possible that in no row and in no column are consecutive numbers written?
