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– level 2

1 Julián writes five positive integers, not necessarily different, such that their product is equal to their sum. What could be the numbers that Julian writes?

2 Pepito's mother wants to prepare n packages of 3 candies to give away at the birthday party, and for this she will buy assorted candies of 3 different flavors. You can buy any number of candies but you can't choose how many of each taste. She wants to put one candy of each flavor in each package, and if this is not possible she will use only candy of one flavor and all the packages will have 3 candies of that flavor. Determine the least number of candies that must be purchased in order to assemble the n packages. He explains why if he buys fewer candies, he is not sure that he will be able to assemble the packages the way he wants.

3 We have a pool table 8 meters long and 2 meters wide with a single ball in the center. We throw the ball in a straight line and, after traveling 29 meters, it stops at a corner of the table. How many times did the ball hit the edges of the table?

Note: When the ball rebounds on the edge of the table, the two angles that form its trajectory with the edge of the table are the same.

4 Find all the natural numbers x, y, z that satisfy simultaneously

$$\begin{cases} xyz = 4104 \\ x + y + z = 77 \end{cases}$$

5 On a 9×9 board, divided into 1×1 squares, pieces of the form
 Each piece covers exactly 3 squares.

(a) Starting from the empty board, what is the maximum number of pieces that can be placed?

(b) Starting from the board with 3 pieces already placed as shown in the diagram below, what is the maximum number of pieces that can be placed?

<https://cdn.artofproblemsolving.com/attachments/d/4/3bd010828accb2d1811d49eb17fa69662ff60.gif>

– level 1

1 Javier multiplies four digits, not necessarily different, and obtains a number ending in 7. Determine how much the sum of the four digits that Javier multiplies can be worth. Give all the possibilities.

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- 2 Inside an 11×11 square, Pablo drew a rectangle and extending its sides divided the square into 5 rectangles, as shown in the figure.

<https://cdn.artofproblemsolving.com/attachments/5/a/7774da7085f283b3aae74fb5ff47257257182.gif>

Sofia did the same, but she also managed to make the lengths of the sides of the 5 rectangles be whole numbers between 1 and 10, all different. Show a figure like the one Sofia made.

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- 3 In each square of a 5×5 board is written 1 or -1 . In each step, the number of each of the 25 cells is replaced by the result of the multiplication of the numbers of all its neighboring cells. Initially we have the board of the figure.

<https://cdn.artofproblemsolving.com/attachments/2/d/29b8e5df29526630102ac400a3a2b2f8fee41.gif>

Show how the board looks after 2004 steps.

Clarification: Two squares are *neighbors* if they have a common side.

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- 4 In a square $ABCD$ of diagonals AC and BD , we call O at the center of the square. A square $PQRS$ is constructed with sides parallel to those of $ABCD$ with P in segment AO , Q in segment BO , R in segment CO , S in segment DO . If area of $ABCD$ equals two times the area of $PQRS$, and M is the midpoint of the AB side, calculate the measure of the angle $\angle AMP$.

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- 5 There are 90 cards and two different digits are written on each one: 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 12, and so on up to 98. A set of cards is *correct* if it does not contain any cards whose first digit is the same as the second digit of another card in the set. We call the *value* of a set of cards the sum of the numbers written on each card. For example, the four cards 04, 35, 78 and 98 form a correct set and their value is 215, since $04 + 35 + 78 + 98 = 215$. Find a correct set that has the largest possible value. Explain why it is impossible to achieve a correct set of higher value.
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