www.artofproblemsolving.com/community/c3146307
by parmenides51

– level 2

1 Four digits a, b, c, d, different from each other and different from zero, are chosen and the list of all the four-digit numbers that are obtained by exchanging the digits a, b, c, d is written. What digits must be chosen so that the list has the greatest possible number of four-digit numbers that are multiples of 36?

2 Let $ABCD$ be a rectangle of sides $AB = 4$ and $BC = 3$. The perpendicular on the diagonal BD drawn from A cuts BD at point H. We call M the midpoint of BH and N the midpoint of CD. Calculate the measure of the segment MN.

3 Find all pairs of positive integers (a, b) such that $8b + 1$ is a multiple of a and $8a + 1$ is a multiple of b.

4 Bob plotted 2003 green points on the plane, so all triangles with three green vertices have area less than 1.
Prove that the 2003 green points are contained in a triangle T of area less than 4.

5 An ant, which is on an edge of a cube of side 8, must travel on the surface and return to the starting point. It's path must contain interior points of the six faces of the cube and should visit only once each face of the cube. Find the length of the path that the ant can carry out and justify why it is the shortest path.

– level 1

1 Pedro writes all the numbers with four different digits that can be made with digits a, b, c, d, that meet the following conditions:

$$a \neq 0\,,\; b = a + 2\,,\; c = b + 2\,,\; d = c + 2$$

Find the sum of all the numbers Pedro wrote.

2 The triangle ABC is right in A and R is the midpoint of the hypotenuse BC . On the major leg AB the point P is marked such that $CP = BP$ and on the segment BP the point Q is marked such that the triangle PQR is equilateral. If the area of triangle ABC is 27, calculate the area of triangle PQR .

3 Find the smallest positive integer that ends in 56, is a multiple of 56, and has the sum of its digits equal to 56.

4 Celia chooses a number n and writes the list of natural numbers from 1 to n: $1, 2, 3, 4, ..., n-1, n$. At each step, it changes the list: it copies the first number to the end and deletes the first two. After $n-1$ steps a single number will be written.
For example, for $n = 6$ the five steps are:

$$1, 2, 3, 4, 5, 6 \to 3, 4, 5, 6, 1 \to 5, 6, 1, 3 \to 1, 3, 5 \to 5, 1 \to 5$$

and the number 5 is written.
Celia chose a number n between 1000 and 3000 and after $n-1$ steps the number 1 was written. Determine all the values of n that Celia could have chosen.
Justify why those values work, and the others do not.

5 We have a 4×4 squared board. We define the *separation* between two squares as the least number of moves that a chess knight must take to go from one square to the other (using moves of the knight). Three boxes A, B, C form a good trio if the three separations between A and B, between A and C and between B and C are equal. Determines the number of good trios that are formed on the board.

Clarification: In each move the knight moves 2 squares in the horizontal direction plus one square in the vertical direction or moves 2 squares in the vertical direction plus one square in the horizontal direction.