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– level 2

1 Using white cubes of side 1, a prism (without holes) was assembled. The faces of the prism were painted black. It is known that the cubes left with exactly 4 white faces are 20 in total. Determine what the dimensions of the prism can be. Give all the possibilities.

2 Let k be a fixed positive integer, $k \leq 10$. Given a list of ten numbers, the allowed operation is: choose k numbers from the list, and add 1 to each of them. Thus, a new list of ten numbers is obtained. If you initially have the list 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, determine the values of k for which it is possible, through a sequence of allowed operations, to obtain a list that has the ten equal numbers. In each case indicate the sequence.

3 In a triangle ABC , right in A and isosceles, let D be a point on the side AC ($A \neq D \neq C$) and E be the point on the extension of BA such that the triangle ADE is isosceles. Let P be the midpoint of segment BD , R be the midpoint of the segment CE and Q the intersection point of ED and BC . Prove that the quadrilateral $ARQP$ is a square

4 The vertices of a regular 2002-sided polygon are numbered 1 through 2002, clockwise. Given an integer n , $1 \leq n \leq 2002$, color vertex n blue, then, going clockwise, count n vertices starting at the next of n , and color n blue. And so on, starting from the vertex that follows the last vertex that was colored, n vertices are counted, colored or uncolored, and the number n is colored blue. When the vertex to be colored is already blue, the process stops. We denote $P(n)$ to the set of blue vertices obtained with this procedure when starting with vertex n . For example, $P(364)$ is made up of vertices 364, 728, 1092, 1456, 1820, 182, 546, 910, 1274, 1638, and 2002. Determine all integers n , $1 \leq n \leq 2002$, such that $P(n)$ has exactly 14 vertices,

5 Let x and y be positive integers we have a table $x \times y$ where $(x + 1)(y + 1)$ points are red (the points are the vertices of the squares). Initially there is one ant in each red point, in a moment the ants walk by the lines of the table with same speed, each turn that an ant arrive in a red point the ant moves 90 to some direction. Determine all values of x and y where is possible that the ants move indefinitely where can't be in any moment two ants in the same red point.

– level 1

1 A group of men, some of them accompanied by their wives, spent \$1.000 on a hotel. Each man spent \$19 and each woman \$13. Determine how many women and how many men there were.

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- 2** A rectangular sheet of paper (white on one side and gray on the other) was folded three times, as shown in the figure:

Rectangle 1, which was white after the first fold, has 20 cm more perimeter than rectangle 2, which was white after the second fold, and this in turn has 16 cm more perimeter than rectangle 3, which was white after the third fold. Determine the area of the sheet.

<https://cdn.artofproblemsolving.com/attachments/d/f/8e363b40654ad0d8e100eac38319ee3784a7a.png>

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- 3** Mustafa bought a big rug. The seller measured the rug with a ruler that was supposed to measure one meter. As it turned out to be 30 meters long by 20 meters wide, he charged Rs 120.000 Rs. When Mustafa arrived home, he measured the rug again and realized that the seller had overcharged him by 9.408 Rs. How many centimeters long is the ruler used by the seller?

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- 4** In a bank, only the manager knows the safe's combination, which is a five-digit number. To support this combination, each of the bank's ten employees is given a five-digit number. Each of these backup numbers has in one of the five positions the same digit as the combination and in the other four positions a different digit than the one in that position in the combination. Backup numbers are: 07344, 14098, 27356, 36429, 45374, 52207, 63822, 70558, 85237, 97665. What is the combination to the safe?

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- 5** Find the maximum number of $3 \times 5 \times 7$ boxes that can be placed inside a $11 \times 35 \times 39$ box. For the number found, indicate how you would place that number of boxes inside the box.
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