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by parmenides51

– level 2

1 In my calculator, one of the keys from 1 to 9 does not work properly: when you press it, a digit between 1 and 9 appears on the screen that is not the correct one. When I tried to write the number 987654321, a number divisible by 11 appeared on the screen and leaves a remainder of 3 when divided by 9. What is the broken key? What is the number that appeared on the screen?

2 On the trapezoid $ABCD$, side DA is perpendicular to the bases AB and CD . The base AB measures 45, the base CD measures 20 and the BC side measures 65. Let P on the BC side such that BP measures 45 and M is the midpoint of DA. Calculate the measure of the PM segment.

3 In a board with 3 rows and 555 columns, 3 squares are colored red, one in each of the 3 rows.
If the numbers from 1 to 1665 are written in the boxes, in row order, from left to right (in the first row from 1 to 555, in the second from 556 to 1110 and in the third from 1111 to 1665) there are 3 numbers that are written in red squares.
If they are written in the boxes, ordered by columns, from top to bottom, the numbers from 1 to 1665 (in the first column from 1 to 3, in the second from 4 to 6, in the third from 7 to 9,... ., and in the last one from 1663 to 1665) there are 3 numbers that are written in red boxes.
We call *red* numbers those that in one of the two distributions are written in red boxes.
Indicate which are the 3 squares that must be colored red so that there are only 3 red numbers.
Show all the possibilities.

4 Ten coins of 1 cm radius are placed around a circle as indicated in the figure.
Each coin is tangent to the circle and its two neighboring coins.
Prove that the sum of the areas of the ten coins is twice the area of the circle.
`https://cdn.artofproblemsolving.com/attachments/5/e/edf7a7d39d749748f4ae818853cb3f8b2b35b`
`gif`

5 On the board are written the natural numbers from 1 to 2001 inclusive. You have to delete some numbers so that among those that remain undeleted it is impossible to choose two different numbers such that the result of their multiplication is equal to one of the numbers that remain undeleted. What is the minimum number of numbers that must be deleted? For that amount, present an example showing which numbers are erased. Justify why, if fewer numbers are deleted, the desired property is not obtained.

– level 1

1 Sara wrote on the board an integer with less than thirty digits and ending in 2. Celia erases the 2 from the end and writes it at the beginning. The number that remains written is equal to twice the number that Sara had written. What number did Sara write?

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2 Let's take a $ABCD$ rectangle of paper; the side AB measures 5 cm and the side BC measures 9 cm.
We do three folds:
1.We take the AB side on the BC side and call P the point on the BC side that coincides with A.
A right trapezoid $BCDQ$ is then formed.
2. We fold so that B and Q coincide. A 5-sided polygon $RPCDQ$ is formed.
3. We fold again by matching D with C and Q with P. A new right trapezoid $RPCS$.
After making these folds, we make a cut perpendicular to SC by its midpoint T, obtaining the right trapezoid $RUTS$.
Calculate the area of the figure that appears as we unfold the last trapezoid $RUTS$.

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3 There are three boxes, one blue, one white and one red, and 8 balls. Each of the balls has a number from 1 to 8 written on it, without repetitions. The 8 balls are distributed in the boxes, so that there are at least two balls in each box. Then, in each box, add up all the numbers written on the balls it contains. The three outcomes are called the blue sum, the white sum, and the red sum, depending on the color of the corresponding box. Find all possible distributions of the balls such that the red sum equals twice the blue sum, and the red sum minus the white sum equals the white sum minus the blue sum.

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4 Using only prime numbers, a set is formed with the following conditions:
Any one-digit prime number can be in the set.
For a prime number with more than one digit to be in the set, the number that results from deleting only the first digit and also the number that results from deleting only the last digit must be in the set.
Write, of the sets that meet these conditions, the one with the greatest number of elements.
Justify why there cannot be one with more elements.

Remember that the number 1 is not prime.

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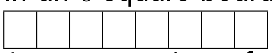
5 In an 8-square board -like the one in the figure- there is initially one checker in each square.



A move consists of choosing two tokens and moving one of them one square to the right and the other one one square to the left. If after 4 moves the 8 checkers are distributed in only 2 boxes, determine what those boxes can be and how many checkers are in each one.
