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– level 2

1 The set $\{1,2,3,4\}$ can be partitioned into two subsets $A = \{1,4\}$ and $B = \{3,2\}$ with no common elements and such that the sum of the elements of A is equal to the sum of the elements of B. Such a partition is impossible for the set $\{1,2,3,4,5\}$ and also for the set $\{1,2,3,4,5,6\}$. Determine all values of n for which the set of the first n natural numbers can be partitioned into two subsets with no common elements such that the sum of the elements of each subset is the same.

2 Given a parallelogram with area 1 and we will construct lines where this lines connect a vertex with a midpoint of the side no adjacent to this vertex; with the 8 lines formed we have a octagon inside of the parallelogram. Determine the area of this octagon

3 Let S be a circle with radius 2, let S_1 be a circle,with radius 1 and tangent, internally to S in B and let S_2 be a circle, with radius 1 and tangent to S_1 in A, but S_2 isn't tangent to S. If K is the point of intersection of the line AB and the circle S, prove that K is in the circle S_2.

4 There is a cube of $3\times3\times3$ formed by the union of 27 cubes of $1\times1\times1$. Some cubes are removed in such a way that those that remain continue to form a solid made up of cubes that are united by at least one facing the rest of the solid. When a cube is removed, those that remain do so in the same place they were. What is the maximum number of cubes that can be removed so that the area of the resulting solid is equal to the area of the original cube?

5 A rectangle with area n with n positive integer, can be divided in n squares(this squares are equal) and the rectangle also can be divided in $n+98$ squares (the squares are equal). Find the sides of this rectangle

– level 1

1 Find all four-digit natural numbers formed by two even digits and two odd digits that verify that when multiplied by 2 four-digit numbers are obtained with all their even digits and when divided by 2 four-digit natural numbers are obtained with all their odd digits.

2 Let ABC be a right triangle in A , whose leg measures 1 cm. The bisector of the angle BAC cuts the hypotenuse in R, the perpendicular to AR on R , cuts the side AB at its midpoint. Find the measurement of the side AB .

3 To write all consecutive natural numbers from $1ab$ to $ab2$ inclusive, $1ab1$ digits have been used. Determine how many more digits are needed to write the natural numbers up to aab inclusive. Give all chances. (a and b represent digits)

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4 There are pieces in the shape of an equilateral triangle with sides $1, 2, 3, 4, 5$ and 6 (50 pieces of each size). You want to build an equilateral triangle of side 7 using some of these pieces, without gaps or overlaps. What is the least number of pieces needed?

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5 In a row there are 12 cards that can be of three kinds: with both white faces, with both black faces or with one white face and the other black. Initially there are 9 cards with the black side facing up. The first six cards from the left are turned over, leaving 9 cards with the black face up. The six cards on the left are then turned over, leaving 8 cards with the black face up. Finally, six cards are turned over: the first three on the left and the last three on the right, leaving 3 cards with the black face up. Decide if with this information it is possible to know with certainty how many cards of each kind are in the row

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