Hong Kong Team Selection Test 2023

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– Test 1

Problem 1 Mandy needs to wake up early for attending a mathematics contest. She has set an alarm in her smartphone every 15 minutes since 5:30 am. If an alarm is not pressed off by her or her mother (or anything else), it will ring for a while, stop for a while, then will ring again 9 minutes later as the first ring, and so on (e.g. if the first alarm is not pressed off, it will ring again at 5:39 am). Also each alarm will work independently. Now suppose each ring-tone lasts for x minutes, and the smartphone has eventually rung for 50 minutes before Mandy wakes up at 6:30 am (assume no one has pressed off any alarm before that). Find the value of x.

Problem 2 Let n be a positive integer. Show that if p is prime dividing $5^{4n}-5^{3n}+5^{2n}-5^n+1$, then $p\equiv 1 \pmod 4$.

Problem 3 A point P lies inside an equilateral triangle ABC such that $AP=15$ and $BP=8$. Find the maximum possible value of the sum of areas of triangles ABP and BCP.

Problem 4 Let x,y,z be real numbers such that $x+y+z\neq 0$. Find the minimum value of $\frac{|x|+|x+4y|+|y+7z|+2|z|}{|x+y+z|}$

Problem 5 Let $ABCD$ be a convex quadrilateral with $AB=5$, $AD=17$, and $CD=6$. If the angle bisector of $\angle BAD$ and $\angle ADC$ intersect at the midpoint of BC, find the area of $ABCD$.

Problem 6 (a) Find the smallest number of lines drawn on the plane so that they produce exactly 2022 points of intersection. (Note: For 1 point of intersection, the minimum is 2; for 2 points, minimum is 3; for 3 points, minimum is 3; for 4 points, minimum is 4; for 5 points, the minimum is 4, etc.)
(b) What happens if the lines produce exactly 2023 intersections?

– Test 2

Problem 1 Given a 24×24 square grid, initially all its unit squares are coloured white. A move consists of choosing a row, or a column, and changing the colours of all its unit squares, from white to black, and from black to white. Is it possible that after finitely many moves, the square grid contains exactly 574 black unit squares?

Problem 2 Giiven ΔABC, $\angle CAB=75^\circ$ and $\angle ACB=45^\circ$. BC is extended to T so that $BC=CT$. Let M be the midpoint of the segment AT. Find $\angle BMC$.

Problem 3 Let $n \geq 4$ be a positive integer. Consider any set A formed by n distinct real numbers such that the following condition holds: for every $a \in A$, there exist distinct elements $x, y, z \in A$ such that $|x-a|, |y-a|, |z-a| \geq 1$. For each n, find the greatest real number M such that $\sum_{a \in A} |a| \geq M$.

Problem 4 A two digit number s is special if s is the common two leading digits of the decimal expansion of 4^n and 5^n, where n is a certain positive integer. Given that there are two special number, find these two special numbers.

Test 3 CHKMO

Problem 1 Suppose a, b and c are nonzero real numberss satisfying $abc = 2$. Prove that among the three numbers $2a - \frac{1}{b}$, $2b - \frac{1}{c}$ and $2c - \frac{1}{a}$, at most two of them are greater than 2.

Problem 2 Find the period of the repetend of the fraction $\frac{39}{1428}$ by using *binary* numbers, i.e. its binary decimal representation.

(Note: When a proper fraction is expressed as a decimal number (of any base), either the decimal number terminates after finite steps, or it is of the form $0.b_1b_2\cdots b_sa_1a_2\cdots a_ka_1a_2\cdots a_ka_1a_2\cdots a_k\cdots$. Here the repeated sequence $a_1a_2\cdots a_k$ is called the *repetend* of the fraction, and the smallest length of the repetend, k, is called the *period* of the decimal number.)

Problem 3 Given a 2023×2023 square grid, there are beetles on some of the unit squares, with at most one beetle on each unit square. In the first minute, every beetle will move one step to its right or left adjacent square. In the second minute, every beetle will move again, only this time, in case the beetle moved right or left in the previous minute, it moves to top or bottom in this minute, and vice versa, and so on. What is the minimum number of beetles on the square grid to ensure that, no matter where the beetles are initially, and how they move in the first minute, but after finitely many minutes, at least two beetles will meet at a certain unit square.

Problem 4 Let $ABCD$ be a quadrilateral inscribed in a circle Γ such that $AB = BC = CD$. Let M and N be the midpoints of AD and AB respectively. The line CM meets Γ again at E. Prove that the tangent at E to Γ, the line AD and the line CN are concurrent.
