Mediterranean Mathematics Olympiad 2022

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by parmenides51

1 Let $S=\{1,...,999\}$. Determine the smallest integer $m.$ for which there exist m two-sided cards C_1,..., C_m with the following properties: • Every card C_i has an integer from S on one side and another integer from S on the other side. • For all $x,y\in S$ with $x\ne y$, it is possible to select a card C_i that shows x on one of its sides and another card C_j (with $i\ne j$) that shows y on one of its sides.

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2 (a) Decide whether there exist two decimal digits a and b, such that every integer with decimal representation $ab222...231$ is divisible by 73.
(b) Decide whether there exist two decimal digits c and d, such that every integer with decimal representation $cd222...231$ is divisible by 79.

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3 Let a,b,c,d be four positive real numbers. Prove that

$$\frac{(a+b+c)^2}{a^2+b^2+c^2}+\frac{(b+c+d)^3}{b^3+c^3+d^3}+\frac{(c+d+a)^4}{c^4+d^4+a^4}+\frac{(d+a+b)^5}{d^5+a^5+b^5}\le 120$$

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4 The triangle ABC is inscribed in a circle γ of center O, with $AB<AC$. A point D on the angle bisector of $\angle BAC$ and a point E on segment BC satisfy OE is parallel to AD and $DE\perp BC$. Point K lies on the extension line of EB such that $EA=EK$. A circle pass through points A,K,D meets the extension line of BC at point P, and meets the circle of center O at point $Q\ne A$. Prove that the line PQ is tangent to the circle γ.
