

**Silk Road Mathematics Competition 2022**

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by Hopeooooo

- 1 Convex quadrilateral  $ABCD$  is inscribed in circle  $w$ . Rays  $AB$  and  $DC$  intersect at  $K$ .  $L$  is chosen on the diagonal  $BD$  so that  $\angle BAC = \angle DAL$ .  $M$  is chosen on the segment  $KL$  so that  $CM \parallel BD$ . Prove that line  $BM$  touches  $w$ .  
(Kungozhin M.)

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- 2 Distinct positive integers  $A$  and  $B$  are given. Prove that there exist infinitely many positive integers that can be represented both as  $x_1^2 + Ay_1^2$  for some positive coprime integers  $x_1$  and  $y_1$ , and as  $x_2^2 + By_2^2$  for some positive coprime integers  $x_2$  and  $y_2$ .  
(Golovanov A.S.)

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- 3 In an infinite sequence  $\{\alpha\}, \{\alpha^2\}, \{\alpha^3\}, \dots$  there are finitely many distinct values. Show that  $\alpha$  is an integer. ( $\{x\}$  denotes the fractional part of  $x$ .)  
(Golovanov A.S.)

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- 4 In a language, an alphabet with 25 letters is used; words are exactly all sequences of (not necessarily different) letters of length 17. Two ends of a paper strip are glued so that the strip forms a ring; the strip bears a sequence of  $5^{18}$  letters. Say that a word is singular if one can cut a piece bearing exactly that word from the strip, but one cannot cut out two such non-overlapping pieces. It is known that one can cut out  $5^{16}$  non-overlapping pieces each containing the same word. Determine the largest possible number of singular words.  
(Bogdanov I.)