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–	Day 1
1	Given is an equilateral triangle ABC with circumcenter O. Let D be a point on to minor arc BC of its circumcircle such that $DB > DC$. The perpendicular bisector of OD meets the circumcircle at E, F, with E lying on the minor arc BC. The lines BE and CF meet at P. Prove that $PD \perp BC$.
2	Let $S = \{13, 133, \cdots\}$ be the set of the positive integers of the form $133\cdots3$. Consider a horizontal row of 2022 cells. Ana and Borja play a game: they alternatively write a digit on the leftmost empty cell, starting with Ana. When the row is filled, the digits are read from left to right to obtain a 2022-digit number N. Borja wins if N is divisible by a number in S, otherwise Ana wins. Find which player has a winning strategy and describe it.
3	Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(yf(x)) + f(x-1) = f(x)f(y)$ and $\lvert f(x)\rvert < 2022$ for all $0 < x < 1$.
–	Day 2
4	Let $n > 2$ be a positive integer. Given is a horizontal row of n cells where each cell is painted blue or red. We say that a block is a sequence of consecutive boxes of the same color. Arepito the crab is initially standing at the leftmost cell. On each turn, he counts the number m of cells belonging to the largest block containing the square he is on, and does one of the following: If the square he is on is blue and there are at least m squares to the right of him, Arepito moves m squares to the right; If the square he is in is red and there are at least m squares to the left of him, Arepito moves m cells to the left; In any other case, he stays on the same square and does not move any further. For each n, determine the smallest integer k for which there is an initial coloring of the row with k blue cells, for which Arepito will reach the rightmost cell.
5	Let ABC be an acute triangle with circumcircle Γ. Let P and Q be points in the half plane defined by BC containing A, such that BP and CQ are tangents to Γ and $PB = BC = CQ$. Let K and L be points on the external bisector of the angle $\angle CAB$, such that $BK = BA, CL = CA$. Let M be the intersection point of the lines PK and QL. Prove that $MK = ML$.

6 Find all functions $f:\mathbb{N}\rightarrow\mathbb{N}$, such that $f(a)f(a+b)-ab$ is a perfect square for all $a,b\in\mathbb{N}$.