Austria Beginners' Competition 2000

www.artofproblemsolving.com/community/c3164185

by parmenides51

1   Let a be a real number. Determine, for all a, all pairs (x, y) of real numbers such that $(x - y^2)(y - x^2) + x^3 + y^3 = a$.

2   Let a, b positive real numbers. Prove that

$$\frac{(a + b)^3}{a^2 b} \geq \frac{27}{4}.$$

When does equality occur?

3   A two-digit number is *nice* if it is both a multiple of the product of its digits and a multiple of the sum of its digits. How many numbers satisfy this property? What is the ratio of the number to the sum of digits for each of the nice numbers?

4   Let $ABCDEFG$ be half of a regular dodecahedron. Let P be the intersection of the lines AB and GF, and let Q be the intersection of the lines AC and GE. Prove that Q is the circumcenter of the triangle AGP.