Regional Competition For Advanced Students 2000

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by parmenides51

1 For which natural numbers n does $2^n > 10n^2 - 60n + 80$ hold?

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2 For any real number a, find all real numbers x that satisfy the following equation.

$$(2x+1)^4 + ax(x+1) - \frac{x}{2} = 0$$

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3 We consider two circles $k_1(M_1, r_1)$ and $k_2(M_2, r_2)$ with $z = M_1M_2 > r_1 + r_2$ and a common outer tangent with the tangent points P_1 and $P2$ (that is, they lie on the same side of the connecting line M_1M_2). We now change the radii so that their sum is $r_1 + r_2 = c$ remains constant. What set of points does the midpoint of the tangent segment P_1P_2 run through, when r_1 varies from 0 to c?

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4 We consider the sequence $\{u_n\}$ defined by recursion $u_{n+1} = \frac{u_n(u_n+1)}{n}$ for $n \geq 1$.
(a) Determine the terms of the sequence for $u_1 = 1$.
(b) Show that if a member of the sequence is rational, then all subsequent members are also rational numbers.
(c) Show that for every natural number K there is a $u_1 > 1$ such that the first K terms of the sequence are natural numbers.
