

Austrian Mathematical Olympiad Junior Regional Competition 2022

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by parmenides51

- 1 Show that for all real numbers x and y with $x > -1$ and $y > -1$ and $x + y = 1$ the inequality

$$\frac{x}{y+1} + \frac{y}{x+1} \geq \frac{2}{3}$$

holds. When does equality apply?

(Walther Janous)

- 2 You are given a rectangular playing field of size 13×2 and any number of dominoes of sizes 2×1 and 3×1 . The playing field should be seamless with such dominoes and without overlapping, with no domino protruding beyond the playing field may. Furthermore, all dominoes must be aligned in the same way, i. e. their long sides must be parallel to each other. How many such coverings are possible?

(Walther Janous)

- 3 A semicircle is erected over the segment AB with center M . Let P be one point different from A and B on the semicircle and Q the midpoint of the arc of the circle AP . The point of intersection of the straight line BP with the parallel to PQ through M is S . Show that $PM = PS$ holds.

(Karl Czakler)

- 4 Determine all prime numbers p, q and r with $p + q^2 = r^4$.

(Karl Czakler)
