Regional Competition For Advanced Students 2022
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by parmenides51

1 Let a and b be positive real numbers with $a^2+b^2=\frac{1}{2}$. Prove that

$$\frac{1}{1-a}+\frac{1}{1-b}\geq 4.$$

When does equality hold?

(Walther Janous)

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2 Determine the number of ten-digit positive integers with the following properties: • Each of the digits $0, 1, 2, ..., 8$ and 9 is contained exactly once. • Each digit, except 9, has a neighbouring digit that is larger than it.
(Note. For example, in the number 1230, the digits 1 and 3 are the neighbouring digits of 2 while 2 and 0 are the neighbouring digits of 3. The digits 1 and 0 have only one neighbouring digit.)

(Karl Czakler)

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3 Let ABC denote a triangle with $AC\neq BC$. Let I and U denote the incenter and circumcenter of the triangle ABC, respectively. The incircle touches BC and AC in the points D and E, respectively. The circumcircles of the triangles ABC and CDE intersect in the two points C and P. Prove that the common point S of the lines CU and PI lies on the circumcircle of the triangle ABC.

(Karl Czakler)

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4 We are given the set

$$M=\{-2^{2022},-2^{2021},...,-2^2,-2,-1,1,2,2^2,...,2^{2021},2^{2022}\}.$$

Let T be a subset of M, such that neighbouring numbers have the same difference when the elements are ordered by size.
(a) Determine the maximum number of elements that such a set T can contain.
(b) Determine all sets T with the maximum number of elements.

(Walther Janous)
