

**Bosnia and Herzegovina Team Selection Test 2021**

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by parmenides51

- 1 Let  $x, y, z$  be real numbers from the interval  $[0, 1]$ . Determine the maximum value of expression

$$W = y \cdot \sqrt{1-x} + z \cdot \sqrt{1-y} + x \cdot \sqrt{1-z}$$

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- 2 Let  $p > 2$  be a prime number. Prove that there is a permutation  $k_1, k_2, \dots, k_{p-1}$  of numbers  $1, 2, \dots, p-1$  such that the number  $1^{k_1} + 2^{k_2} + 3^{k_3} + \dots + (p-1)^{k_{p-1}}$  is divisible by  $p$ .

Note: The numbers  $k_1, k_2, \dots, k_{p-1}$  are a permutation of the numbers  $1, 2, \dots, p-1$  if each of the numbers  $1, 2, \dots, p-1$  appears exactly once among the numbers  $k_1, k_2, \dots, k_{p-1}$ .

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- 4 An L-shaped figure composed of 4 unit squares (such as shown in the picture) we call L-dominoes. <https://cdn.artofproblemsolving.com/attachments/b/2/064b7c7de496f981cd937cbb7392efc106642.png>

Determine the maximum number of L-dominoes that can be placed on a board of dimensions  $n \times n$ , where  $n$  is natural number, so that no two dominoes overlap and it is possible get from the upper left to the lower right corner of the board by moving only across those squares that are not covered by dominoes. (By moving, we move from someone of the square on it the neighboring square, i.e. the square with which it shares the page).

Note: L-Dominoes can be rotated as well as flipped, giving an symmetrical figure wrt axis compared to the one shown in the picture.

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