## AoPS Community

Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2002
www.artofproblemsolving.com/community/c3168239
by parmenides51

5-6 p1. Find all triples of positive integers such that the sum of their reciprocals is equal to one.
p2. Prove that $a(a+1)(a+2)(a+3)$ is divisible by 24 .
p3. There are 20 very small red chips and some blue ones. Find out whether it is possible to put them on a large circle such that
(a) for each chip positioned on the circle the antipodal position is occupied by a chip of different color;
(b) there are no two neighboring blue chips.
p4. A 12 liter container is filled with gasoline. How to split it in two equal parts using two empty 5 and 8 liter containers?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/ community/c5h2760506p24143309).

7-9 p1. One out of 12 coins is counterfeited. It is known that its weight differs from the weight of a valid coin but it is unknown whether it is lighter or heavier. How to detect the counterfeited coin with the help of four trials using only a two-pan balance without weights?
p2. Below a 3-digit number $c d e$ is multiplied by a 2-digit number $a b$. Find all solutions $a, b, c, d, e, f, g$ if it is known that they represent distinct digits.

|  | $c$ | $d$ | $e$ |
| ---: | ---: | ---: | ---: |
| $x$ |  | $a$ | $b$ |
|  |  | $f$ | $e$ |
| + | $c$ | $d$ | $e$ |
|  | $b$ | $b$ | $c$ |

p3. Find all integer $n$ such that $\frac{n+1}{2 n-1}$ is an integer.
p4. There are several straight lines on the plane which split the plane in several pieces. Is it possible to paint the plane in brown and green such that each piece is painted one color and no pieces having a common side are painted the same color?

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10-12 p1. Find all integer solutions of the equation $a^{2}-b^{2}=2002$.
p2. Prove that the disks drawn on the sides of a convex quadrilateral as on diameters cover this quadrilateral.
p3. 30 students from one school came to Mathematical Olympiad. In how many different ways is it possible to place them in four rooms?
p4. A 12 liter container is filled with gasoline. How to split it in two equal parts using two empty 5 and 8 liter containers?

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