Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2003

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by parmenides51

5-6 **p1.** One day, Granny Smith bought a certain number of apples at Horock's Farm Market. When she returned the next day she found that the price of the apples was reduced by 20%. She could therefore buy more apples while spending the same amount as the previous day. How many percent more?

p2. You are asked to move several boxes. You know nothing about the boxes except that each box weighs no more than 10 tons and their total weight is 100 tons. You can rent several trucks, each of which can carry no more than 30 tons. What is the minimal number of trucks you can rent and be sure you will be able to carry all the boxes at once?

p3. The five numbers $1, 2, 3, 4, 5$ are written on a piece of paper. You can select two numbers and increase them by 1. Then you can again select two numbers and increase those by 1. You can repeat this operation as many times as you wish. Is it possible to make all numbers equal?

p4. There are 15 people in the room. Some of them are friends with others. Prove that there is a person who has an even number of friends in the room.

Bonus Problem

p5. Several ants are crawling along a circle with equal constant velocities (not necessarily in the same direction). If two ants collide, both immediately reverse direction and crawl with the same velocity. Prove that, no matter how many ants and what their initial positions are, they will, at some time, all simultaneously return to the initial positions.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

7-9 **p1**. Is it possible to find n positive numbers such that their sum is equal to 1 and the sum of their squares is less than $\frac{1}{10}$?

p2. In the country of Sepulia, there are several towns with airports. Each town has a certain number of scheduled, round-trip connecting flights with other towns. Prove that there are two towns that have connecting flights with the same number of towns.

p3. A 4×4 magic square is a 4×4 table filled with numbers $1, 2, 3, ..., 16$ - with each number appearing exactly once - in such a way that the sum of the numbers in each row, in each column, and in each diagonal is the same. Is it possible to complete $\begin{bmatrix} 2 & 3 & * & * \\ 4 & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$ to a magic square? (That is, can you replace the stars with remaining numbers $1, 5, 6, ..., 16$, to obtain a magic square?)

p4. Is it possible to label the edges of a cube with the numbers $1, 2, 3, ..., 12$ in such a way that the sum of the numbers labelling the three edges coming into a vertex is the same for all vertices?

p5. (Bonus) Several ants are crawling along a circle with equal constant velocities (not necessarily in the same direction). If two ants collide, both immediately reverse direction and crawl with the same velocity. Prove that, no matter how many ants and what their initial positions are, they will, at some time, all simultaneously return to the initial positions.

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10-12 **p1.** The length of the first side of a triangle is 1, the length of the second side is 11, and the length of the third side is an integer. Find that integer.

p2. Suppose a, b, and c are positive numbers such that $a+b+c = 1$. Prove that $ab+ac+bc \le \frac{1}{3}$.

p3. Prove that $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{100}$ is not an integer.

p4. Find all of the four-digit numbers n such that the last four digits of n^2 coincide with the digits of n.

p5. (Bonus) Several ants are crawling along a circle with equal constant velocities (not necessarily in the same direction). If two ants collide, both immediately reverse direction and crawl with the same velocity. Prove that, no matter how many ants and what their initial positions are, they will, at some time, all simultaneously return to the initial positions.

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