Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2004

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by parmenides51

5-6

p1. On the island of Nevermind some people are liars; they always lie. The remaining habitants of the island are truthlovers; they tell only the truth. Three habitants of the island, A, B, and C met this morning. A said: "All of us are liars". B said: "Only one of us is a truthlover".
Who of them is a liar and who of them is a truthlover?

p2. Pinocchio has 9 pieces of paper. He is allowed to take a piece of paper and cut it in 5 pieces or 7 pieces which increases the number of his pieces. Then he can take again one of his pieces of paper and cut it in 5 pieces or 7 pieces. He can do this again and again as many times as he wishes. Can he get 2004 pieces of paper?

p3. In Dragonland there are coins of 1 cent, 2 cents, 10 cents, 20 cents, and 50 cents. What is the largest amount of money one can have in coins, yet still not be able to make exactly 1 dollar?

p4. Find all solutions a, b, c, d, e if it is known that they represent distinct digits and satisfy the following:

$$\begin{array}{cccccc} & & a & b & c & d \\ + & & a & c & a & c \\ \hline & c & d & e & b & c \end{array}$$

p5. Two players play the following game. On the lowest left square of an 8×8 chessboard there is a rook. The first player is allowed to move the rook up or to the right by an arbitrary number of squares. The second player is also allowed to move the rook up or to the right by an arbitrary number of squares. Then the first player is allowed to do this again, and so on. The one who moves the rook to the upper right square wins. Who has a winning strategy?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

7-9

p1. Two players play the following game. On the lowest left square of an 8×8 chessboard there is a rook. The first player is allowed to move the rook up or to the right by an arbitrary number of squares. The second player is also allowed to move the rook up or to the right by an arbitrary number of squares. Then the first player is allowed to do this again, and so on. The one who moves the rook to the upper right square wins. Who has a winning strategy?

p2. In Crocodile Country there are banknotes of 1 dollar, 10 dollars, 100 dollars, and $1,000$ dollars.

Is it possible to get 1,000,000 dollars by using $250,000$ banknotes?

p3. Fifteen positive numbers (not necessarily whole numbers) are placed around the circle. It is known that the sum of every four consecutive numbers is 30. Prove that each number is less than 15.

p4. Donald Duck has 100 sticks, each of which has length 1 cm or 3 cm. Prove that he can break into 2 pieces no more than one stick, after which he can compose a rectangle using all sticks.

p5. Three consecutive 2 digit numbers are written next to each other. It turns out that the resulting 6 digit number is divisible by 17. Find all such numbers.

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10-12 **p1.** Two players play the following game. On the lowest left square of an 8×8 chessboard there is a rook (castle). The first player is allowed to move the rook up or to the right by an arbitrary number of squares. The second layer is also allowed to move the rook up or to the right by an arbitrary number of squares. Then the first player is allowed to do this again, and so on. The one who moves the rook to the upper right square wins. Who has a winning strategy?

p2. Find the smallest positive whole number that ends with 17, is divisible by 17, and the sum of its digits is 17.

p3. Three consecutive 2-digit numbers are written next to each other. It turns out that the resulting 6-digit number is divisible by 17. Find all such numbers.

p4. Let $ABCD$ be a convex quadrilateral (a quadrilateral $ABCD$ is called convex if the diagonals AC and BD intersect). Suppose that $\angle CBD = \angle CAB$ and $\angle ACD = \angle BDA$. Prove that $\angle ABC = \angle ADC$.

p5. A circle of radius 1 is cut into four equal arcs, which are then arranged to make the shape shown on the picture. What is its area?
`https://cdn.artofproblemsolving.com/attachments/f/3/49c3fe8b218ab0a5378ecc635b797a912723f png`

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