Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2006

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by parmenides51

5-6

p1. Find all solutions a, b, c, d, e, f if it is known that they represent distinct digits and satisfy the following:
$$\begin{array}{cccccc} & & a & b & c & a \\ + & & & d & d & e \\ & & & & d & e \\ \hline & d & f & f & d & d \end{array}$$

p2. Snowhite wrote on a piece of paper a whole number greater than 1 and multiplied it by itself. She obtained a number, all digits of which are 1: $n^2 = 111...111$ Does she know how to multiply?

p3. Two players play the following game on an 8×8 chessboard. The first player can put a bishop on an arbitrary square. Then the second player can put another bishop on a free square that is not controlled by the first bishop. Then the first player can put a new bishop on a free square that is not controlled by the bishops on the board. Then the second player can do the same, etc. A player who cannot put a new bishop on the board loses the game. Who has a winning strategy?

p4. Four girls Marry, Jill, Ann and Susan participated in the concert. They sang songs. Every song was performed by three girls. Mary sang 8 songs, more then anybody. Susan sang 5 songs less then all other girls. How many songs were performed at the concert?

p5. Pinocchio has a 10×10 table of numbers. He took the sums of the numbers in each row and each such sum was positive. Then he took the sum of the numbers in each columns and each such sum was negative. Can you trust Pinocchio's calculations?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

7-9

p1. Find all solutions a, b, c, d, e, f if it is known that they represent distinct digits and satisfy the following:
$$\begin{array}{cccccc} & & a & b & c & a \\ + & & & d & d & e \\ & & & & d & e \\ \hline & d & f & f & d & d \end{array}$$

p2. Explain whether it possible that the sum of two squares of positive whole numbers has all digits equal to 1:

$$n^2 + m^2 = 111...111$$

p3. Two players play the following game on an 8×8 chessboard. The first player can put a rook on an arbitrary square. Then the second player can put another rook on a free square that is not controlled by the first rook. Then the first player can put a new rook on a free square that is not controlled by the rooks on the board. Then the second player can do the same, etc. A player who cannot put a new rook on the board loses the game. Who has a winning strategy?

p4. Show that the difference $9^{2008} - 7^{2008}$ is divisible by 10.

p5. Is it possible to find distict positive whole numbers a, b, c, d, e such that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} = 1?$$

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

10-12 **p1.** A right triangle has hypotenuse of length 12 cm. The height corresponding to the right angle has length 7 cm. Is this possible?
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p2. Prove that from any 5 integers one can choose 3 such that their sum is divisible by 3.

p3. Two players play the following game on an 8×8 chessboard. The first player can put a knight on an arbitrary square. Then the second player can put another knight on a free square that is not controlled by the first knight. Then the first player can put a new knight on a free square that is not controlled by the knights on the board. Then the second player can do the same, etc. A player who cannot put a new knight on the board loses the game. Who has a winning strategy?

p4. Consider a regular octagon $ABCDEGH$ (i.e., all sides of the octagon are equal and all angles of the octagon are equal). Show that the area of the rectangle $ABEF$ is one half of the area of the octagon.
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p5. Can you find a positive whole number such that after deleting the first digit and the zeros following it (if they are) the number becomes 24 times smaller?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).