Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2007

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by parmenides51

5-6   **p1.** The Evergreen School booked buses for a field trip. Altogether, 138 people went to West Lake, while 115 people went to East Lake. The buses all had the same number of seats, and every bus has more than one seat. All seats were occupied and everybody had a seat. How many seats were there in each bus?

p2. In New Scotland there are three kinds of coins: 1 cent, 6 cent, and 36 cent coins. Josh has 50 of the 36-cent coins (and no other coins). He is allowed to exchange a 36 cent coin for 6 coins of 6 cents, and to exchange a 6 cent coin for 6 coins of 1 cent. Is it possible that after several exchanges Josh will have 150 coins?

p3. Pinocchio multiplied two 2 digit numbers. But witch Masha erased some of the digits. The erased digits are the ones marked with a $*$. Could you help Pinocchio to restore all the erased

```
                    9   5
        x           *   *
        _____
                    *   *
    +   1   *   *
    _____
        *   *   *   *
```

digits? Find all solutions.

p4. There are 50 senators and 435 members of House of Representatives. On Friday all of them voted a very important issue. Each senator and each representative was required to vote either "yes" or "no". The announced results showed that the number of "yes" votes was greater than the number of "no" votes by 24. Prove that there was an error in counting the votes.

p5. Was there a year in the last millennium (from 1000 to 2000) such that the sum of the digits of that year is equal to the product of the digits?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

7-9   **p1.** The Evergreen School booked buses for a field trip. Altogether, 138 people went to West Lake, while 115 people went to East Lake. The buses all had the same number of seats and every bus has more than one seat. All seats were occupied and everybody had a seat. How many seats were on each bus?

p2. In New Scotland there are three kinds of coins: 1 cent, 6 cent, and 36 cent coins. Josh has 99 of the 36-cent coins (and no other coins). He is allowed to exchange a 36 cent coin for 6 coins of 6 cents, and to exchange a 6 cent coin for 6 coins of 1 cent. Is it possible that after several exchanges Josh will have 500 coins?

p3. Find all solutions a, b, c, d, e, f, g, h if these letters represent distinct digits and the following multiplication is correct:

```
        a   b   c
    +       d   e
    ───────────────
        f   a   g   c
    x   b   b   h
    ───────────────
    f   f   e   g   c
```

p4. Is it possible to find a rectangle of perimeter 10 m and cut it in rectangles (as many as you want) so that the sum of the perimeters is 500 m?

p5. The picture shows a maze with chambers (shown as circles) and passageways (shown as segments). A cat located in chamber C tries to catch a mouse that was originally in the chamber M. The cat makes the first move, moving from chamber C to one of the neighboring chambers. Then the mouse moves, then the cat, and so forth. At each step, the cat and the mouse can move to any neighboring chamber or not move at all. The cat catches the mouse by moving into the chamber currently occupied by the mouse. Can the cat get the mouse?
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png

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

10-12 **p1.** 17 rooks are placed on an 8×8 chess board. Prove that there must be at least one rook that is attacking at least 2 other rooks.

p2. In New Scotland there are three kinds of coins: 1 cent, 6 cent, and 36 cent coins. Josh has 99 of the 36-cent coins (and no other coins). He is allowed to exchange a 36 cent coin for 6 coins of 6 cents, and to exchange a 6 cent coin for 6 coins of 1 cent. Is it possible that after several exchanges Josh will have 500 coins?

p3. Find all solutions $a, b, c, d, e, f, g, h, i$ if these letters represent distinct digits and the following

multiplication is correct:

```
        a   b   c
   x        d   e
   ─────────────────
     f   a   c   c
   + g   h   i
   ─────────────────
   f   f   f   c   c
```

p4. Pinocchio rode a bicycle for 3.5 hours. During every 1-hour period he went exactly 5 km. Is it true that his average speed for the trip was 5 km/h? Explain your reasoning.

p5. Let a, b, c be odd integers. Prove that the equation $ax^2 + bx + c = 0$ cannot have a rational solution.

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