Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2008

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by parmenides51

5-6 **p1.** Insert "$+$" signs between some of the digits in the following sequence to obtain correct equality:

$$1 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \;\; 6 \;\; 7 = 100$$

p2. A square is tiled by smaller squares as shown in the figure. Find the area of the black square in the middle if the perimeter of the big square $ABCD$ is 40 cm.
https://cdn.artofproblemsolving.com/attachments/8/c/d54925cba07f63ec8578048f46e1e730cb8df
png

p3. Jack made 3 quarts of fruit drink from orange and apple juice. $\frac{2}{5}$ of his drink is orange juice and the rest is apple juice. Nick prefers more orange juice in the drink. How much orange juice should he add to the drink to obtain a drink composed of $\frac{3}{5}$ of orange juice?

p4. A train moving at 55 miles per hour meets and is passed by a train moving moving in the opposite direction at 35 miles per hour. A passenger in the first train sees that the second train takes 8 seconds to pass him. How long is the second train?

p5. It is easy to arrange 16 checkers in 10 rows of 4 checkers each, but harder to arrange 9 checkers in 10 rows of 3 checkers each. Do both.

p6. Every human that lived on Earth exchanged some number of handshakes with other humans. Show that the number of people that made an odd number of handshakes is even.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

7-9 **p1.** Jack made 3 quarts of fruit drink from orange and apple juice. His drink contains 45% of orange juice. Nick prefers more orange juice in the drink. How much orange juice should he add to the drink to obtain a drink composed of 60% of orange juice?

p2. A square is tiled by smaller squares as shown in the figure. Find the area of the black square in the middle if the perimeter of the big square $ABCD$ is 40 cm.

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p3. For one particular number $a > 0$ the function f satisfies the equality $f(x+a) = \frac{1+f(x)}{1-f(x)}$ for all x. Show that f is a periodic function. (A function f is periodic with the period T if $f(x+T) = f(x)$ for any x.)

p4. If a, b, c, x, y, z are numbers so that $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$ and $\frac{a}{x}+\frac{b}{y}+\frac{c}{z} = 0$. Show that $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$

p5. Is it possible that a four-digit number $AABB$ is a perfect square?
(Same letters denote the same digits).

p6. A finite number of arcs of a circle are painted black (see figure). The total length of these arcs is less than $\frac{1}{5}$ of the circumference. Show that it is possible to inscribe a square in the circle so that all vertices of the square are in the unpainted portion of the circle.
https://cdn.artofproblemsolving.com/attachments/2/c/bdfa61917a47f3de5dd3684627792a9ebf05c
png

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

10-12 **p1.** A square is tiled by smaller squares as shown in the figure. Find the area of the black square in the middle if the perimeter of the square $ABCD$ is 14 cm.
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png

p2. If a, b, and c are numbers so that $a + b + c = 0$ and $a^2 + b^2 + c^2 = 1$. Compute $a^4 + b^4 + c^4$.

p3. A given fraction $\frac{a}{b}$ (a, b are positive integers, $a \ne b$) is transformed by the following rule: first, 1 is added to both the numerator and the denominator, and then the numerator and the denominator of the new fraction are each divided by their greatest common divisor (in other words, the new fraction is put in simplest form). Then the same transformation is applied again and again. Show that after some number of steps the denominator and the numerator differ exactly by 1.

p4. A goat uses horns to make the holes in a new 30×60 cm large towel. Each time it makes two new holes. Show that after the goat repeats this 61 times the towel will have at least two holes whose distance apart is less than 6 cm.

p5. You are given 555 weights weighing 1 g, 2 g, 3 g, ... , 555 g. Divide these weights into three groups whose total weights are equal.

p6. Draw on the regular 8×8 chessboard a circle of the maximal possible radius that intersects only black squares (and does not cross white squares). Explain why no larger circle can satisfy the condition.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).