

**Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2009**

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by parmenides51

**5-6 p1.** Anne purchased yesterday at WalMart in Puerto Rico 6 identical notebooks, 8 identical pens and 7 identical erasers. Anne remembers that each eraser costs 73 cents. She did not buy anything else. Anne told her mother that she spent 12 dollars and 76 cents at Walmart. Can she be right? Note that in Puerto Rico there is no sales tax.

**p2.** Two men ski one after the other first in a flat field and then uphill. In the field the men run with the same velocity 12 kilometers/hour. Uphill their velocity drops to 8 kilometers/hour. When both skiers enter the uphill trail segment the distance between them is 300 meters less than the initial distance in the field. What was the initial distance between skiers? (There are 1000 meters in 1 kilometer.)

**p3.** In the equality  $** + **** = ****$  all the digits are replaced by \*. Restore the equality if it is known that any numbers in the equality does not change if we write all its digits in the opposite order.

**p4.** If a polyleg has even number of legs he always tells truth. If he has an odd number of legs he always lies. Once a green polyleg told a dark-blue polyleg "- I have 8 legs. And you have only 6 legs!" The offended dark-blue polyleg replied "-It is me who has 8 legs, and you have only 7 legs!" A violet polyleg added "-The dark-blue polyleg indeed has 8 legs. But I have 9 legs!" Then a stripped polyleg started: "-None of you has 8 legs. Only I have 8 legs!" Which polyleg has exactly 8 legs?

**p5.** Cut the figure shown below in two equal pieces. (Both the area and the form of the pieces must be the same.) <https://cdn.artofproblemsolving.com/attachments/e/4/778678c1e8748e213ffc.png>

PS. You should use hide for answers. Collected here (<https://artofproblemsolving.com/community/c5h2760506p24143309>).

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**7-9 p1.** Arrange the whole numbers 1 through 15 in a row so that the sum of any two adjacent numbers is a perfect square. In how many ways this can be done?

**p2.** Prove that if  $p$  and  $q$  are prime numbers which are greater than 3 then  $p^2 - q^2$  is divisible by 24.

**p3.** If a polyleg has even number of legs he always tells truth. If he has an odd number of legs he always lies.

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Which polyleg has exactly 8 legs?

**[b]p4.**[/b] There is a small puncture (a point) in the wall (as shown in the figure below to the right). The housekeeper has a small flag of the following form (see the figure left). Show on the figure all the points of the wall where you can hammer in a nail such that if you hang the flag it will close up the puncture.

<https://cdn.artofproblemsolving.com/attachments/a/f/8bb55a3fdfb0aff8e62bc4cf20a2d3436f5d9.png>

**p5.** Assume  $a, b, c$  are odd integers. Show that the quadratic equation  $ax^2 + bx + c = 0$  has no rational solutions.

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**10-12 p1.** Compute the sum of sharp angles at all five nodes of the star below.

(figure missing ([http://www.math.msu.edu/~mshapiro/NewOlympiad/Olymp2009/10\\_12\\_2009.pdf](http://www.math.msu.edu/~mshapiro/NewOlympiad/Olymp2009/10_12_2009.pdf)))

**p2.** Arrange the integers from 1 to 15 in a row so that the sum of any two consecutive numbers is a perfect square. In how many ways this can be done?

**p3.** Prove that if  $p$  and  $q$  are prime numbers which are greater than 3 then  $p^2 - q^2$  is divisible by 24.

**p4.** A city in a country is called Large Northern if comparing to any other city of the country it is either larger or farther to the North (or both). Similarly, a city is called Small Southern. We know that in the country all cities are Large Northern city. Show that all the cities in this country are simultaneously Small Southern.

**p5.** You have four tall and thin glasses of cylindrical form. Place on the flat table these four

glasses in such a way that all distances between any pair of centers of the glasses' bottoms are equal.

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