

Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2010

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by parmenides51

5-6 p1. Ben and his dog are walking on a path around a lake. The path is a loop 500 meters around. Suddenly the dog runs away with velocity 10 km/hour. Ben runs after it with velocity 8 km/hour. At the moment when the dog is 250 meters ahead of him, Ben turns around and runs at the same speed in the opposite direction until he meets the dog. For how many minutes does Ben run?

p2. The six interior angles in two triangles are measured. One triangle is obtuse (i.e. has an angle larger than 90°) and the other is acute (all angles less than 90°). Four angles measure 120° , 80° , 55° and 10° . What is the measure of the smallest angle of the acute triangle?

p3. The figure below shows a 10×10 square with small 2×2 squares removed from the corners. What is the area of the shaded region?

<https://cdn.artofproblemsolving.com/attachments/7/5/a829487cc5d937060e8965f6da3f4744ba558.png>

p4. Two three-digit whole numbers are called relatives if they are not the same, but are written using the same triple of digits. For instance, 244 and 424 are relatives. What is the minimal number of relatives that a three-digit whole number can have if the sum of its digits is 10?

p5. Three girls, Ann, Kelly, and Kathy came to a birthday party. One of the girls wore a red dress, another wore a blue dress, and the last wore a white dress. When asked the next day, one girl said that Kelly wore a red dress, another said that Ann did not wear a red dress, the last said that Kathy did not wear a blue dress. One of the girls was truthful, while the other two lied. Which statement was true?

PS. You should use hide for answers. Collected here (<https://artofproblemsolving.com/community/c5h2760506p24143309>).

7-9 p1. Find the smallest whole number $n \geq 2$ such that the product $(2^2 - 1)(3^2 - 1)\dots(n^2 - 1)$ is the square of a whole number.

p2. The figure below shows a 10×10 square with small 2×2 squares removed from the corners. What is the area of the shaded region?

<https://cdn.artofproblemsolving.com/attachments/7/5/a829487cc5d937060e8965f6da3f4744ba558.png>

p3. Three cars are racing: a Ford [F], a Toyota [T], and a Honda [H]. They began the race with F first, then T , and H last. During the race, F was passed a total of 3 times, T was passed 5 times, and H was passed 8 times. In what order did the cars finish?

p4. There are 11 big boxes. Each one is either empty or contains 8 medium-sized boxes inside. Each medium box is either empty or contains 8 small boxes inside. All small boxes are empty. Among all the boxes, there are a total of 102 empty boxes. How many boxes are there altogether?

p5. Ann, Mary, Pete, and finally Vlad eat ice cream from a tub, in order, one after another. Each eats at a constant rate, each at his or her own rate. Each eats for exactly the period of time that it would take the three remaining people, eating together, to consume half of the tub. After Vlad eats his portion there is no more ice cream in the tube. How many times faster would it take them to consume the tub if they all ate together?

PS. You should use hide for answers. Collected here (<https://artofproblemsolving.com/community/c5h2760506p24143309>).

10-12 p1. Find all solutions a, b, c, d, e, f, g if it is known that they represent distinct digits and satisfy

the following:

$$\begin{array}{rcccccc}
 & & & & a & b & c & d \\
 x & & & & & & & a & b \\
 \hline
 & & c & d & b & d & b & & \\
 + & c & e & b & f & b & & & \\
 \hline
 & c & g & a & e & g & b & &
 \end{array}$$

p2. 5 numbers are placed on the circle. It is known that the sum of any two neighboring numbers is not divisible by 3 and the sum of any three consecutive numbers is not divisible by 3. How many numbers on the circle are divisible by 3?

p3. n teams played in a volleyball tournament. Each team played precisely one game with all other teams. If x_j is the number of victories and y_j is the number of losses of the j th team, show that

$$\sum_{j=1}^n x_j^2 = \sum_{j=1}^n y_j^2$$

p4. Three cars participated in the car race: a Ford [F], a Toyota [T], and a Honda [H]. They began the race with F first, then T , and H last. During the race, F was passed a total of 3 times, T was passed 5 times, and H was passed 8 times. In what order did the cars finish?

p5. The side of the square is 4 cm. Find the sum of the areas of the six half-disks shown on the

picture.

<https://cdn.artofproblemsolving.com/attachments/c/b/73be41b9435973d1c53a20ad2eb436b1384d6.png>

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