Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2012

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5-6 **p1.** A boy has as many sisters as brothers. How ever, his sister has twice as many brothers as sisters. How many boys and girls are there in the family?

p2. Solve each of the following problems.
(1) Find a pair of numbers with a sum of 11 and a product of 24.
(2) Find a pair of numbers with a sum of 40 and a product of 400.
(3) Find three consecutive numbers with a sum of 333.
(4) Find two consecutive numbers with a product of 182.

p3. 2008 integers are written on a piece of paper. It is known that the sum of any 100 numbers is positive. Show that the sum of all numbers is positive.

p4. Let p and q be prime numbers greater than 3. Prove that $p^2 - q^2$ is divisible by 24.

p5. Four villages A, B, C, and D are connected by trails as shown on the map.
`https://cdn.artofproblemsolving.com/attachments/4/9/33ecc416792dacba65930caa61adbae09b829`
`png`
On each path $A \to B \to C$ and $B \to C \to D$ there are 10 hills, on the path $A \to B \to D$ there are 22 hills, on the path $A \to D \to B$ there are 45 hills. A group of tourists starts from A and wants to reach D. They choose the path with the minimal number of hills. What is the best path for them?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

7-9 **p1.** We say that integers a and b are *friends* if their product is a perfect square. Prove that if a is a friend of b, then a is a friend of $gcd(a, b)$.

p2. On the island of knights and liars, a traveler visited his friend, a knight, and saw him sitting at a round table with five guests.
”I wonder how many knights are among you?” he asked.
” Ask everyone a question and find out yourself” advised him one of the guests.
”Okay. Tell me one: Who are your neighbors?” asked the traveler.
This question was answered the same way by all the guests.

”This information is not enough!” said the traveler.
”But today is my birthday, do not forget it!” said one of the guests.
”Yes, today is his birthday!” said his neighbor.
Now the traveler was able to find out how many knights were at the table.
Indeed, how many of them were there if *knights always tell the truth and liars always lie*?

p3. A rope is folded in half, then in half again, then in half yet again. Then all the layers of the rope were cut in the same place. What is the length of the rope if you know that one of the pieces obtained has length of 9 meters and another has length 4 meters?

p4. The floor plan of the palace of the Shah is a square of dimensions 6×6, divided into rooms of dimensions 1×1. In the middle of each wall between rooms is a door. The Shah orders his architect to eliminate some of the walls so that all rooms have dimensions 2×1, no new doors are created, and a path between any two rooms has no more than N doors. What is the smallest value of N such that the order could be executed?

p5. There are 10 consecutive positive integers written on a blackboard. One number is erased. The sum of remaining nine integers is 2011. Which number was erased?

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10-12 **p1.** A triangle ABC is drawn in the plane. A point D is chosen inside the triangle. Show that the sum of distances $AD + BD + CD$ is less than the perimeter of the triangle.

p2. In a triangle ABC the bisector of the angle C intersects the side AB at M, and the bisector of the angle A intersects CM at the point T. Suppose that the segments CM and AT divided the triangle ABC into three isosceles triangles. Find the angles of the triangle ABC.

p3. You are given 100 weights of masses $1, 2, 3, ..., 99, 100$. Can one distribute them into 10 piles having the following property: the heavier the pile, the fewer weights it contains?

p4. Each cell of a 10×10 table contains a number. In each line the greatest number (or one of the largest, if more than one) is underscored, and in each column the smallest (or one of the smallest) is also underscored. It turned out that all of the underscored numbers are underscored exactly twice. Prove that all numbers stored in the table are equal to each other.

p5. Two stores have warehouses in which wheat is stored. There are 16 more tons of wheat in the first warehouse than in the second. Every night exactly at midnight the owner of each store steals from his rival, taking a quarter of the wheat in his rival's warehouse and dragging it to his own. After 10 days, the thieves are caught. Which warehouse has more wheat at this point and by how much?

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