Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2013

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by parmenides51

5-6 **p1.** The clock is 2 hours 20 minutes ahead of the correct time each week. The clock is set to the correct time at midnight Sunday to Monday. What time does this clock show at 6pm correct time on Thursday?

p2. Five cities A, B, C, D, and E are located along the straight road in the alphabetical order. The sum of distances from B to A, C, D and E is 20 miles. The sum of distances from C to the other four cities is 18 miles. Find the distance between B and C.

p3. Does there exist distinct digits a, b, c, and d such that $\overline{abc} + \overline{c} = \overline{bda}$?
Here $\overline{abc}$ means the three digit number with digits a, b, and c.

p4. Kuzya, Fyokla, Dunya, and Senya participated in a mathematical competition. Kuzya solved 8 problems, more than anybody else. Senya solved 5 problem, less than anybody else. Each problem was solved by exactly 3 participants. How many problems were there?

p5. Mr Mouse got to the cellar where he noticed three heads of cheese weighing 50 grams, 80 grams, and 120 grams. Mr. Mouse is allowed to cut simultaneously 10 grams from any two of the heads and eat them. He can repeat this procedure as many times as he wants. Can he make the weights of all three pieces equal?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

7-9 **p1.** A straight line is painted in two colors. Prove that there are three points of the same color such that one of them is located exactly at the midpoint of the interval bounded by the other two.

p2. Find all positive integral solutions x, y of the equation $xy = x + y + 3$.

p3. Can one cut a square into isosceles triangles with angle 80^o between equal sides?

p4. 20 children are grouped into 10 pairs: one boy and one girl in each pair. In each pair the boy is taller than the girl. Later they are divided into pairs in a different way. May it happen now that

(a) in all pairs the girl is taller than the boy;
(b) in 9 pairs out of 10 the girl is taller than the boy?

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10-12 **p1.** A function f defined on the set of positive numbers satisfies the equality

$$f(xy) = f(x) + f(y), x, y > 0.$$

Find $f(2007)$ if $f\left(\frac{1}{2007}\right) = 1$.

p2. The plane is painted in two colors. Show that there is an isosceles right triangle with all vertices of the same color.

p3. Show that the number of ways to cut a $2n \times 2n$ square into 1×2 dominoes is divisible by 2.

p4. Two mirrors form an angle. A beam of light falls on one mirror. Prove that the beam is reflected only finitely many times (even if the angle between mirrors is very small).

p5. A sequence is given by the recurrence relation $a_{n+1} = (s(a_n))^2 + 1$, where $s(x)$ is the sum of the digits of the positive integer x. Prove that starting from some moment the sequence is periodic.

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