Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2014

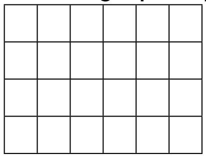
www.artofproblemsolving.com/community/c3168254

by parmenides51

5-6

p1. Find any integer solution of the puzzle: $WE + ST + RO + NG = 128$
(different letters mean different digits between 1 and 9).

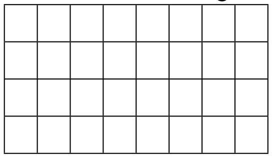
p2. A 5×6 rectangle is drawn on the piece of graph paper (see the figure below). The side of each square on the graph paper is 1 cm long. Cut the rectangle along the sides of the graph squares in two parts whose areas are equal but perimeters are different by 2 cm.



p3. Three runners started simultaneously on a 1 km long track. Each of them runs the whole distance at a constant speed. Runner A is the fastest. When he runs 400 meters then the total distance run by runners B and C together is 680 meters. What is the total combined distance remaining for runners B and C when runner A has 100 meters left?

p4. There are three people in a room. Each person is either a knight who always tells the truth or a liar who always tells lies. The first person said ”We are all liars”. The second replied ”Only you are a liar”. Is the third person a liar or a knight?

p5. A 5×8 rectangle is divided into forty 1×1 square boxes (see the figure below). Choose 24 such boxes and one diagonal in each chosen box so that these diagonals don't have common points.



PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

7-9

p1. (a) Put the numbers 1 to 6 on the circle in such way that for any five consecutive numbers the sum of first three (clockwise) is larger than the sum of remaining two.
(b) Can you arrange these numbers so it works both clockwise and counterclockwise.

p2. A girl has a box with 1000 candies. Outside the box there is an infinite number of chocolates and muffins. A girl may replace: • two candies in the box with one chocolate bar, • two muffins in

the box with one chocolate bar, • two chocolate bars in the box with one candy and one muffin, • one candy and one chocolate bar in the box with one muffin, • one muffin and one chocolate bar in the box with one candy.
Is it possible that after some time it remains only one object in the box?

p3. Find any integer solution of the puzzle: $WE + ST + RO + NG = 128$ (different letters mean different digits between 1 and 9).

p4. Two consecutive three-digit positive integer numbers are written one after the other one. Show that the six-digit number that is obtained is not divisible by 1001.

p5. There are 9 straight lines drawn in the plane. Some of them are parallel some of them intersect each other. No three lines do intersect at one point. Is it possible to have exactly 17 intersection points?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

10-12 **p1.** The length of the side AB of the trapezoid with bases AD and BC is equal to the sum of lengths $|AD| + |BC|$. Prove that bisectors of angles A and B do intersect at a point of the side CD.

p2. Polynomials $P(x) = x^4 + ax^3 + bx^2 + cx + 1$ and $Q(x) = x^4 + cx^3 + bx^2 + ax + 1$ have two common roots. Find these common roots of both polynomials.

p3. A girl has a box with 1000 candies. Outside the box there is an infinite number of chocolates and muffins. A girl may replace: • two candies in the box with one chocolate bar, • two muffins in the box with one chocolate bar, • two chocolate bars in the box with one candy and one muffin, • one candy and one chocolate bar in the box with one muffin, • one muffin and one chocolate bar in the box with one candy.
Is it possible that after some time it remains only one object in the box?

p4. There are 9 straight lines drawn in the plane. Some of them are parallel some of them intersect each other. No three lines do intersect at one point. Is it possible to have exactly 17 intersection points?

p5. It is known that x is a real number such that $x + \frac{1}{x}$ is an integer. Prove that $x^n + \frac{1}{x^n}$ is an integer for any positive integer n.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).