Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2015

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by parmenides51

5-6 **p1.** To every face of a given cube a new cube of the same size is glued. The resulting solid has how many faces?

p2. A father and his son returned from a fishing trip. To make their catches equal the father gave to his son some of his fish. If, instead, the son had given his father the same number of fish, then father would have had twice as many fish as his son. What percent more is the father's catch more than his son's?

p3. A radio transmitter has 4 buttons. Each button controls its own switch: if the switch is OFF the button turns it ON and vice versa. The initial state of switches in unknown. The transmitter sends a signal if at least 3 switches are ON. What is the minimal number of times you have to push the button to guarantee the signal is sent?

p4. 19 matches are placed on a table to show the incorrect equation: $XXX + XIV = XV$. Move exactly one match to change this into a correct equation.

p5. Cut the grid shown into two parts of equal area by cutting along the lines of the grid.
https://cdn.artofproblemsolving.com/attachments/c/1/7f2f284acf3709c2f6b1bea08835d2fb409c4
png

p6. A family of funny dwarfs consists of a dad, a mom, and a child. Their names are: A, R, and C (not in order). During lunch, C made the statements: "R and A have different genders" and "R and A are my parents", and A made the statements "I am C's dad" and "I am R's daughter." In fact, each dwarf told truth once and told a lie once. What is the name of the dad, what is the name of the child, and is the child a son or a daughter?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

7-9 **p1.** Thirty players participate in a chess tournament. Every player plays one game with every other player. What maximal number of players can get exactly 5 points? (any game adds 1 point to the winner's score, 0 points to a loser's score, in the case of a draw each player obtains $1/2$ point.)

p2. A father and his son returned from a fishing trip. To make their catches equal the father gave to his son some of his fish. If, instead, the son had given his father the same number of fish, then father would have had twice as many fish as his son. What percent more is the father's catch more than his son's?

p3. What is the maximal number of pieces of two shapes, https://cdn.artofproblemsolving.com/attachments/a/5/6c567cf6a04b0aa9e998dbae3803b6eeb24a35.png and https://cdn.artofproble
com/attachments/8/a/7a7754d0f2517c93c5bb931fb7b5ae8f5e3217.png, that can be used to tile a 7×7 square?

p4. Six shooters participate in a shooting competition. Every participant has 5 shots. Each shot adds from 1 to 10 points to shooter's score. Every person can score totally for all five shots from 5 to 50 points. Each participant gets 7 points for at least one of his shots. The scores of all participants are different. We enumerate the shooters 1 to 6 according to their scores, the person with maximal score obtains number 1, the next one obtains number 2, the person with minimal score obtains number 6. What score does obtain the participant number 3? The total number of all obtained points is 264.

p5. There are 2014 stones in a pile. Two players play the following game. First, player A takes some number of stones (from 1 to 30) from the pile, then player B takes 1 or 2 stones, then player A takes 2 or 3 stones, then player B takes 3 or 4 stones, then player A takes 4 or 5 stones, etc. The player who gets the last stone is the winner. If no player gets the last stone (there is at least one stone in the pile but the next move is not allowed) then the game results in a draw. Who wins the game using the right strategy?

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10-12 **p1.** What is the maximal number of pieces of two shapes, https://cdn.artofproblemsolving.com/attachments/a/5/6c567cf6a04b0aa9e998dbae3803b6eeb24a35.png and https://cdn.artofproble
com/attachments/8/a/7a7754d0f2517c93c5bb931fb7b5ae8f5e3217.png, that can be used to tile a 7×7 square?

p2. Six shooters participate in a shooting competition. Every participant has 5 shots. Each shot adds from 1 to 10 points to shooter's score. Every person can score totally for all five shots from 5 to 50 points. Each participant gets 7 points for at least one of his shots. The scores of all participants are different. We enumerate the shooters 1 to 6 according to their scores, the person with maximal score obtains number 1, the next one obtains number 2, the person with minimal score obtains number 6. What score does obtain the participant number 3? The total

number of all obtained points is 264.

p2. There are exactly n students in a high school. Girls send messages to boys. The first girl sent messages to 5 boys, the second to 7 boys, the third to 6 boys, the fourth to 8 boys, the fifth to 7 boys, the sixth to 9 boys, the seventh to 8, etc. The last girl sent messages to all the boys. Prove that n is divisible by 3.

p4. In what minimal number of triangles can one cut a 25×12 rectangle in such a way that one can tile by these triangles a 20×15 rectangle.

p5. There are 2014 stones in a pile. Two players play the following game. First, player A takes some number of stones (from 1 to 30) from the pile, then player B takes 1 or 2 stones, then player A takes 2 or 3 stones, then player B takes 3 or 4 stones, then player A takes 4 or 5 stones, etc. The player who gets the last stone is the winner. If no player gets the last stone (there is at least one stone in the pile but the next move is not allowed) then the game results in a draw. Who wins the game using the right strategy?

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