Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2017

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5-6

p1. Replace $*$'s by an arithmetic operations (addition, subtraction, multiplication or division) to obtain true equality
$$2 * 0 * 1 * 6 * 7 = 1.$$

p2. The interval of length 88 cm is divided into three unequal parts. The distance between middle points of the left and right parts is 46 cm. Find the length of the middle part.

p3. A 5×6 rectangle is drawn on a square grid. Paint some cells of the rectangle in such a way that every 3×2 sub-rectangle has exactly two cells painted.

p4. There are 8 similar coins. 5 of them are counterfeit. A detector can analyze any set of coins and show if there are counterfeit coins in this set. The detector neither determines which coins nare counterfeit nor how many counterfeit coins are there. How to run the detector twice to find for sure at least one counterfeit coin?

p5. There is a set of 20 weights of masses $1, 2, 3, ...$ and 20 grams. Can one divide this set into three groups of equal total masses?

p6. Replace letters A, B, C, D, E, F, G by the digits $0, 1, ..., 9$ to get true equality $AB + CD = EF * EG$ (different letters correspond to different digits, same letter means the same digit, AB, CD, EF, and EG are two-digit numbers).

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

7-9

p1. There are 5 weights of masses $1, 2, 3, 5$, and 10 grams. One of the weights is counterfeit (its weight is different from what is written, it is unknown if the weight is heavier or lighter). How to find the counterfeit weight using simple balance scales only twice?

p2. There are 998 candies and chocolate bars and 499 bags. Each bag may contain two items (either two candies, or two chocolate bars, or one candy and one chocolate bar). Ann distributed candies and chocolate bars in such a way that half of the candies share a bag with a chocolate bar. Helen wants to redistribute items in the same bags in such a way that half of the chocolate bars would share a bag with a candy. Is it possible to achieve that?

p3. Insert in sequence 2222222222 arithmetic operations and brackets to get the number 999 (For instance, from the sequence 22222 one can get the number 45: $22 * 2 + 2/2 = 45$).

p4. Put numbers from 15 to 23 in a 3×3 table in such a way to make all sums of numbers in two neighboring cells distinct (neighboring cells share one common side).

p5. All integers from 1 to 200 are colored in white and black colors. Integers 1 and 200 are black, 11 and 20 are white. Prove that there are two black and two white numbers whose sums are equal.

p6. Show that 38 is the sum of few positive integers (not necessarily, distinct), the sum of whose reciprocals is equal to 1. (For instance, $11 = 6 + 3 + 2$, $1/16 + 1/13 + 1/12 = 1$.)

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10-12 **p1.** In the group of five people any subgroup of three persons contains at least two friends. Is it possible to divide these five people into two subgroups such that all members of any subgroup are friends?

p2. Coefficients a, b, c in expression $ax^2 + bx + c$ are such that $b - c > a$ and $a \neq 0$. Is it true that equation $ax^2 + bx + c = 0$ always has two distinct real roots?

p3. Point D is a midpoint of the median AF of triangle ABC. Line CD intersects AB at point E. Distances $|BD| = |BF|$. Show that $|AE| = |DE|$.

p4. Real numbers a, b satisfy inequality $a + b^5 > ab^5 + 1$. Show that $a + b^7 > ba^7 + 1$.

p5. A positive number was rounded up to the integer and got the number that is bigger than the original one by 28%. Find the original number (find all solutions).

p6. Divide a 5×5 square along the sides of the cells into 8 parts in such a way that all parts are different.

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