Mid-Michigan Mathematical Olympiad, Grades 5-6, 7-9 and 10-12 for 2019

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by parmenides51

5-6 **p1.** It takes 12 months for Santa Claus to pack gifts. It would take 20 months for his apprentice to do the job. If they work together, how long will it take for them to pack the gifts?

p2. All passengers on a bus sit in pairs. Exactly $2/5$ of all men sit with women, exactly $2/3$ of all women sit with men. What part of passengers are men?

p3. There are 100 colored balls in a box. Every 10-tuple of balls contains at least two balls of the same color. Show that there are at least 12 balls of the same color in the box.

p4. There are 81 wheels in storage marked by their two types, say first and second type. Wheels of the same type weigh equally. Any wheel of the second type is much lighter than a wheel of the first type. It is known that exactly one wheel is marked incorrectly. Show that one can determine which wheel is incorrectly marked with four measurements.

p5. Remove from the figure below the specified number of matches so that there are exactly 5 squares of equal size left:
(a) 8 matches
(b) 4 matches
`https://cdn.artofproblemsolving.com/attachments/4/b/0c5a65f2d9b72fbea50df12e328c024a0c788.png`

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

7-9 **p1.** Prove that the equation $x^6 - 143x^5 - 917x^4 + 51x^3 + 77x^2 + 291x + 1575 = 0$ has no integer solutions.

p2. There are 81 wheels in a storage marked by their two types, say first and second type. Wheels of the same type weigh equally. Any wheel of the second type is much lighter than a wheel of the first type. It is known that exactly one wheel is marked incorrectly. Show that it can be detected with certainty after four measurements on a balance scale.

p3. Rob and Ann multiplied the numbers from 1 to 100 and calculated the sum of digits of this

product. For this sum, Rob calculated the sum of its digits as well. Then Ann kept repeating this operation until he got a one-digit number. What was this number?

p4. Rui and Jui take turns placing bishops on the squares of the 8×8 chessboard in such a way that bishops cannot attack one another. (In this game, the color of the rooks is irrelevant.) The player who cannot place a rook loses the game. Rui takes the first turn. Who has a winning strategy, and what is it?

p5. The following figure can be cut along sides of small squares into several (more than one) identical shapes. What is the smallest number of such identical shapes you can get?
https://cdn.artofproblemsolving.com/attachments/8/e/9cd09a04209774dab34bc7f989b79573453f3.png

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10-12 **p1.** In triangle ABC, the median BM is drawn. The length $|BM| = |AB|/2$. The angle $\angle ABM = 50^o$. Find the angle $\angle ABC$.

p2. Is there a positive integer n which is divisible by each of $1, 2, 3, ..., 2018$ except for two numbers whose difference is 7?

p3. Twenty numbers are placed around the circle in such a way that any number is the average of its two neighbors. Prove that all of the numbers are equal.

p4. A finite number of frogs occupy distinct integer points on the real line. At each turn, a single frog jumps by 1 to the right so that all frogs again occupy distinct points. For some initial configuration, the frogs can make n moves in m ways. Prove that if they jump by 1 to the left (instead of right) then the number of ways to make n moves is also m.

p5. A square box of chocolates is divided into 49 equal square cells, each containing either dark or white chocolate. At each move Alex eats two chocolates of the same kind if they are in adjacent cells (sharing a side or a vertex). What is the maximal number of chocolates Alex can eat regardless of distribution of chocolates in the box?

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