

Kettering University Mathematics Olympiad For High School Students

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- Today was the 5th Kettering Olympiad - and here are the problems, which are very good intermediate problems.
 1. Find all real x so that $(1 + x^2)(1 + x^4) = 4x^3$
 2. Mark and John play a game. They have 100 pebbles on a table. They take turns taking at least one at at most eight pebbles away. The person to claim the last pebble wins. Mark goes first. Can you find a way for Mark to always win? What about John?
 3. Prove that
$$\sin x + \sin 3x + \sin 5x + \dots + \sin 11x = (1 - \cos 12x)/(2 \sin x)$$
 4. Mark has 7 pieces of paper. He takes some of them and splits each into 7 pieces of paper. He repeats this process some number of times. He then tells John he has 2000 pieces of paper. John tells him he is wrong. Why is John right?
 5. In a triangle ABC , the altitude, angle bisector, and median split angle A into four equal angles. Find the angles of ABC .
 6. There are 100 cities. There exist airlines connecting pairs of cities.
 - a) Find the minimal number of airlines such that with at most k plane changes, one can go from any city to any other city.
 - b) Given that there are 4852 airlines, show that, given any schematic, one can go from any city to any other city.