

**Kettering University Mathematics Olympiad For High School Students**

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- **p1.** Prove that if  $a, b, c, d$  are real numbers, then

$$\max\{a + c, b + d\} \leq \max\{a, b\} + \max\{c, d\}$$

- p2.** Find the smallest positive integer whose digits are all ones which is divisible by 3333333.

- p3.** Find all integer solutions of the equation  $\sqrt{x} + \sqrt{y} = \sqrt{2560}$ .

- p4.** Find the irrational number:

$$A = \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \dots + \frac{1}{2} \sqrt{\frac{1}{2}}}}$$

( $n$  square roots).

- p5.** The Math country has the shape of a regular polygon with  $N$  vertexes.  $N$  airports are located on the vertexes of that polygon, one airport on each vertex. The Math Airlines company decided to build  $K$  additional new airports inside the polygon. However the company has the following policies:

- (i) it does not allow three airports to lie on a straight line,
- (ii) any new airport with any two old airports should form an isosceles triangle.

How many airports can be added to the original  $N$ ?

- p6.** The area of the union of the  $n$  circles is greater than  $9 \text{ m}^2$  (some circles may have non-empty intersections). Is it possible to choose from these  $n$  circles some number of non-intersecting circles with total area greater than  $1 \text{ m}^2$ ?

PS. You should use hide for answers.