Kettering University Mathematics Olympiad For High School Students

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by parmenides51

– **p1.** Solve the equation $x^2 - x - cosy + 1.25 = 0$.

p2. Solve the inequality: $\left|\frac{x-2}{x-3}\right| \le x$

p3. Bilbo and Dwalin are seated at a round table of radius R. Bilbo places a coin of radius r at the center of the table, then Dwalin places a second coin as near to the table's center as possible without overlapping the first coin. The process continues with additional coins being placed as near as possible to the center of the table and in contact with as many coins as possible without overlap. The person who places the last coin entirely on the table (no overhang) wins the game. Assume that R/r is an integer.
(a) Who wins, Bilbo or Dawalin? Please justify your answer.
(b) How many coins are on the table when the game ends?

p4. In the center of a square field is an orc. Four elf guards are on the vertices of that square. The orc can run in the field, the elves only along the sides of the square. Elves run 1.5 times faster than the orc. The orc can kill one elf but cannot fight two of them at the same time. Prove that elves can keep the orc from escaping from the field.

p5. Nine straight roads cross the Mirkwood which is shaped like a square, with an area of 120 square miles. Each road intersects two opposite sides of the square and divides the Mirkwood into two quadrilaterals of areas 40 and 80 square miles. Prove that there exists a point in the Mirkwood which is an intersection of at least three roads.

PS. You should use hide for answers.

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