Kettering University Mathematics Olympiad For High School Students

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- **p1.** At 8 AM Black Widow and Hawkeye began to move towards each other from two cities. They were planning to meet at the midpoint between two cities, but because Black Widow was driving 100 mi/h faster than Hawkeye, they met at the point that is located 120 miles from the midpoint. When they met Black Widow said "If I knew that you drive so slow I would have started one hour later, and then we would have met exactly at the midpoint". Find the distance between cities.

  p2. Solve the inequality: $\frac{x-1}{x-2} \le \frac{x-2}{x-1}$.

  p3. Solve the equation: $(x - y - z)^2 + (2x - 3y + 2z + 4)^2 + (x + y + z - 8)^2 = 0$.

  p4. Three camps are located in the vertices of an equilateral triangle. The roads connecting camps are along the sides of the triangle. Captain America is inside the triangle and he needs to know the distances between camps. Being able to see the roads he has found that the sum of the shortest distances from his location to the roads is 50 miles. Can you help Captain America to evaluate the distances between the camps.

  p5. N regions are located in the plane, every pair of them have a nonempty overlap. Each region is a connected set, that means every two points inside the region can be connected by a curve all points of which belong to the region. Iron Man has one charge remaining to make a laser shot. Is it possible for him to make the shot that goes through all N regions?

  p6. Numbers $1, 2, ..., 100$ are randomly divided in two groups 50 numbers in each. In the first group the numbers are written in increasing order and denoted $a_1, a_2, ... , a_{50}$. In the second group the numbers are written in decreasing order and denoted $b_1, b_2, ..., b_{50}$. Thus, $a_1 < a_2 < ... < a_{50}$ and $b_1 > b2_> ... > b_{50}$. Evaluate $|a_1 - b_1| + |a_2 - b_2| + ... + |a_{50} - b_{50}|$.

  PS. You should use hide for answers.