Kettering University Mathematics Olympiad For High School Students

www.artofproblemsolving.com/community/c3168277

by parmenides51

- **p1.** Darth Vader urgently needed a new Death Star battle station. He sent requests to four planets asking how much time they would need to build it. The Mandalorians answered that they can build it in one year, the Sorganians in one and a half year, the Nevarroins in two years, and the Klatoonians in three years. To expedite the work Darth Vader decided to hire all of them to work together. The Rebels need to know when the Death Star is operational. Can you help the Rebels and find the number of days needed if all four planets work together? We assume that one year $= 365$ days.

  p2. Solve the inequality: $\left(\sin \frac{\pi}{12}\right)^{\sqrt{1-x}} > \left(\sin \frac{\pi}{12}\right)^{x}$

  p3. Solve the equation: $\sqrt{x^2 + 4x + 4} = x^2 + 3x - 6$

  p4. Solve the system of inequalities on $[0, 2\pi]$:

  $$\sin(2x) \geq \sin(x)$$

  $$\cos(2x) \leq \cos(x)$$

  p5. The planet Naboo is under attack by the imperial forces. Three rebellian camps are located at the vertices of a triangle. The roads connecting the camps are along the sides of the triangle. The length of the first road is less than or equal to 20 miles, the length of the second road is less than or equal to 30 miles, and the length of the third road is less than or equal to 45 miles. The Rebels have to cover the area of this triangle by a defensive field. What is the maximal area that they may need to cover?

  p6. The Lake Country on the planet Naboo has the shape of a square. There are nine roads in the country. Each of the roads is a straight line that divides the country into two trapezoidal parts such that the ratio of the areas of these parts is $2 : 5$. Prove that at least three of these roads intersect at one point.

  PS. You should use hide for answers.