individual oral math Olympiad by University of Washington , Grades 6-7 and 8-10

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6-7 Round 1

p1. Is it possible to draw some number of diagonals in a convex hexagon so that every diagonal crosses EXACTLY three others in the interior of the hexagon? (Diagonals that touch at one of the corners of the hexagon DO NOT count as crossing.)

p2. A 3×3 square grid is filled with positive numbers so that
(a) the product of the numbers in every row is 1,
(b) the product of the numbers in every column is 1,
(c) the product of the numbers in any of the four 2×2 squares is 2.
What is the middle number in the grid? Find all possible answers and show that there are no others.

p3. Each letter in $HAGRID$'s name represents a distinct digit between 0 and 9. Show that

$$HAGRID \times H \times A \times G \times R \times I \times D$$

is divisible by 3. (For example, if $H = 1$, $A = 2$, $G = 3$, $R = 4$, $I = 5$, $D = 64$, then $HAGRID \times H \times A \times G \times R \times I \times D = 123456 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6$).

p4. You walk into a room and find five boxes sitting on a table. Each box contains some number of coins, and you can see how many coins are in each box. In the corner of the room, there is a large pile of coins. You can take two coins at a time from the pile and place them in different boxes. If you can add coins to boxes in this way as many times as you like, can you guarantee that each box on the table will eventually contain the same number of coins?

p5. Alex, Bob and Chad are playing a table tennis tournament. During each game, two boys are playing each other and one is resting. In the next game the boy who lost a game goes to rest, and the boy who was resting plays the winner. By the end of tournament, Alex played a total of 10 games, Bob played 15 games, and Chad played 17 games. Who lost the second game?

Round 2

p6. After going for a swim in his vault of gold coins, Scrooge McDuck decides he wants to try to arrange some of his gold coins on a table so that every coin he places on the table touches exactly three others. Can he possibly do this? You need to justify your answer. (Assume the gold coins are circular, and that they all have the same size. Coins must be laid at on the table, and no two of them can overlap.)

p7. You have a deck of 50 cards, each of which is labeled with a number between 1 and 25. In the deck, there are exactly two cards with each label. The cards are shuffled and dealt to 25 students who are sitting at a round table, and each student receives two cards. The students will now play a game. On every move of the game, each student takes the card with the smaller number out of his or her hand and passes it to the person on his/her right. Each student makes this move at the same time so that everyone always has exactly two cards. The game continues until some student has a pair of cards with the same number. Show that this game will eventually end.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

8-10 Round 1

p1. In the convex quadrilateral $ABCD$ with diagonals AC and BD, you know that angle BAC is congruent to angle CBD, and that angle ACD is congruent to angle ADB. Show that angle ABC is congruent to angle ADC.
`https://cdn.artofproblemsolving.com/attachments/5/d/41cd120813d5541dc73c5d4a6c86cc82747fc png`

p2. In how many different ways can you place 12 chips in the squares of a 4×4 chessboard so that
(a) there is at most one chip in each square, and
(b) every row and every column contains exactly three chips.

p3. Students from Hufflepuff and Ravenclaw were split into pairs consisting of one student from each house. The pairs of students were sent to Honeydukes to get candy for Father's Day. For each pair of students, either the Hufflepuff student brought back twice as many pieces of candy as the Ravenclaw student or the Ravenclaw student brought back twice as many pieces of candy as the Hufflepuff student. When they returned, Professor Trelawney determined that the students had brought back a total of 1000 pieces of candy. Could she have possibly been right? Why or why not? Assume that candy only comes in whole pieces (cannot be divided into parts).

p4. While you are on a hike across Deception Pass, you encounter an evil troll, who will not let you across the bridge until you solve the following puzzle. There are six stones, two colored

red, two colored yellow, and two colored green. Aside from their colors, all six stones look and feel exactly the same. Unfortunately, in each colored pair, one stone is slightly heavier than the other. Each of the lighter stones has the same weight, and each of the heavier stones has the same weight. Using a balance scale to make TWO measurements, decide which stone of each color is the lighter one.

p5. Alex, Bob and Chad are playing a table tennis tournament. During each game, two boys are playing each other and one is resting. In the next game the boy who lost a game goes to rest, and the boy who was resting plays the winner. By the end of tournament, Alex played a total of 10 games, Bob played 15 games, and Chad played 17 games. Who lost the second game?

Round 2

p6. Consider a set of finitely many points on the plane such that if we choose any three points A, B, C from the set, then the area of the triangle ABC is less than 1. Show that all of these points can be covered by a triangle whose area is less than 4.

p7. A palindrome is a number that is the same when read forward and backward. For example, 1771 and 23903030932 are palindromes. Can the number obtained by writing the numbers from 1 to n in order be a palindrome for some $n > 1$? (For example, if $n = 11$, the number obtained is 1234567891011, which is not a palindrome.)

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