individual oral math Olympiad by University of Washington , Grades 6-7 and 8-10

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by parmenides51

6-7 Round 1

p1. In a chemical lab there are three vials: one that can hold 1 oz of fluid, another that can hold 2 oz, and a third that can hold 3 oz. The first is filled with grape juice, the second with sulfuric acid, and the third with water. There are also 3 empty vials in the cupboard, also of sizes 1 oz, 2 oz, and 3 oz. In order to save the world with grape-flavored acid, James Bond must make three full bottles, one of each size, filled with a mixture of all three liquids so that each bottle has the same ratio of juice to acid to water. How can he do this, considering he was silly enough not to bring any equipment?

p2. Twelve people, some are knights and some are knaves, are sitting around a table. Knaves always lie and knights always tell the truth. At some point they start up a conversation. The first person says, "There are no knights around this table." The second says, "There is at most one knight at this table." The third – "There are at most two knights at the table." And so on until the 12th says, "There are at most eleven knights at the table." How many knights are at the table? Justify your answer.

p3. Aquaman has a barrel divided up into six sections, and he has placed a red herring in each. Aquaman can command any fish of his choice to either 'jump counterclockwise to the next sector' or 'jump clockwise to the next sector.' Using a sequence of exactly 30 of these commands, can he relocate all the red herrings to one sector? If yes, show how. If no, explain why not.
`https://cdn.artofproblemsolving.com/attachments/0/f/956f64e346bae82dee5cbd1326b0d1789100f png`

p4. Is it possible to place 13 integers around a circle so that the sum of any 3 adjacent numbers is exactly 13?

p5. Two girls are playing a game. The first player writes the letters A or B in a row, left to right, adding one letter on her turn. The second player switches any two letters after each move by the first player (the letters do not have to be adjacent), or does nothing, which also counts as a move. The game is over when each player has made 2011 moves. Can the second player plan her moves so that the resulting letters form a palindrome? (A palindrome is a sequence that reads the same forward and backwards, e.g. $AABABAA$.)

Round 2

p6. Eight students participated in a math competition. There were eight problems to solve. Each problem was solved by exactly five people. Show that there are two students who solved all eight problems between them.

p7. There are $3n$ checkers of three different colors: n red, n green and n blue. They were used to randomly fill a board with 3 rows and n columns so that each square of the board has one checker on it. Prove that it is possible to reshuffle the checkers within each row so that in each column there are checkers of all three colors. Moving checkers to a different row is not allowed.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

8-10 Round 1

p1. Twelve people, some are knights and some are knaves, are sitting around a table.
Knaves always lie and knights always tell the truth. At some point they start up a conversation.
The first person says, ”There are no knights around this table.”
The second says, ”There is at most one knight at this table.”
The third – ”There are at most two knights at the table.”
And so on until the 12th says, ”There are at most eleven knights at the table.”
How many knights are at the table? Justify your answer.

p2. Show that in the sequence 10017, 100117, 1001117, $...$ all numbers are divisible by 53.

p3. Harry and Draco have three wands: a bamboo wand, a willow wand, and a cherry wand, all of the same length. They must perform a spell wherein they take turns picking a wand and breaking it into three parts – first Harry, then Draco, then Harry again. But in order for the spell to work, Harry has to make sure it is possible to form three triangles out of the pieces of the wands, where each triangle has a piece from each wand. How should he break the wands to ensure the success of the spell?

p4. A $2 \times 2 \times 2$ cube has 4 equal squares on each face. The squares that share a side are called neighbors (thus, each square has 4 neighbors – see picture). Is it possible to write an integer in each square in such a way that the sum of each number with its 4 neighbors is equal to 13? If yes, show how. If no, explain why not.
`https://cdn.artofproblemsolving.com/attachments/8/4/0f7457f40be40398dee806d125ba26780f9d3 png`

p5. Two girls are playing a game. The first player writes the letters A or B in a row, left to right, adding one letter on her turn. The second player switches any two letters after each move by the first player (the letters do not have to be adjacent), or does nothing, which also counts as a move. The game is over when each player has made 2011 moves. Can the second player plan her moves so that the resulting letters form a palindrome? (A palindrome is a sequence that reads the same forward and backwards, e.g. $AABABAA$.)

Round 2

p6. A red square is placed on a table. 2010 white squares, each the same size as the red square, are then placed on the table in such a way that the red square is fully covered and the sides of every white square are parallel to the sides of the red square. Is it always possible to remove one of the white squares so the red square remains completely covered?

p7. A computer starts with a given positive integer to which it randomly adds either 54 or 77 every second and prints the resulting sum after each addition. For example, if the computer is given the number 1, then a possible output could be: 1, 55, 109, 186, $\dots$ Show that after finitely many seconds the computer will print a number whose last two digits are the same.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).