individual oral math Olympiad by University of Washington , Grades 5-7 and 8-10

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by parmenides51

5-7   Round 1

p1. Tom and Jerry stole a chain of 7 sausages and are now trying to divide the bounty. They take turns biting the sausages at one of the connections. When one of them breaks a connection, he may eat any single sausages that may fall out. Tom takes the first bite. Each of them is trying his best to eat more sausages than his opponent. Who will succeed?

p2. The King of the Mountain Dwarves wants to light his underground throne room by placing several torches so that the whole room is lit. The king, being very miserly, wants to use as few torches as possible. What is the least number of torches he could use? (You should show why he can't do it with a smaller number of torches.)
This is the shape of the throne room:
https://cdn.artofproblemsolving.com/attachments/b/2/719daafd91fc9a11b8e147bb24cb66b7a684e
png
Also, the walls in all rooms are lined with velvet and do not reflect the light. For example, the picture on the right shows how another room in the castle is partially lit.
https://cdn.artofproblemsolving.com/attachments/5/1/0f6971274e8c2ff3f2d0fa484b567ff3d631f
png

p3. In the Hundred Acre Wood, all the animals are either knights or liars. Knights always tell the truth and liars always lie. One day in the Wood, Winnie-the-Pooh, a knight, decides to visit his friend Rabbit, also a noble knight. Upon arrival, Pooh finds his friend sitting at a round table with 5 other guests.
One-by-one, Pooh asks each person at the table how many of his two neighbors are knights. Surprisingly, he gets the same answer from everybody! "Oh bother!" proclaims Pooh. "I still don't have enough information to figure out how many knights are at this table."
"But it's my birthday," adds one of the guests. "Yes, it's his birthday!" agrees his neighbor.
Now Pooh can tell how many knights are at the table. Can you?

p4. Several girls participate in a tennis tournament in which each player plays each other player exactly once. At the end of the tournament, it turns out that each player has lost at least one of her games. Prove that it is possible to find three players A, B, and C such that A defeated B, B defeated C, and C defeated A.

p5. There are 40 piles of stones with an equal number of stones in each. Two players, Ann and Bob, can select any two piles of stones and combine them into one bigger pile, as long as this pile would not contain more than half of all the stones on the table. A player who can't make a move loses. Ann goes first. Who wins?

Round 2

p6. In a galaxy far, far away, there is a United Galactic Senate with 100 Senators. Each Senator has no more than three enemies. Tired of their arguments, the Senators want to split into two parties so that each Senator has no more than one enemy in his own party. Prove that they can do this. (Note: If A is an enemy of B, then B is an enemy of A.)

p7. Harry has a 2012 by 2012 chessboard and checkers numbered from 1 to 2012×2012. Can he place all the checkers on the chessboard in such a way that whatever row and column Professor Snape picks, Harry will be able to choose three checkers from this row and this column such that the product of the numbers on two of the checkers will be equal to the number on the third?
`https://cdn.artofproblemsolving.com/attachments/b/3/a87d559b340ceefee485f41c8fe44ae9a5911`
`png`

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

8-10   Round 1

p1. In the Hundred Acre Wood, all the animals are either knights or liars. Knights always tell the truth and liars always lie. One day in the Wood, Winnie-the-Pooh, a knight, decides to visit his friend Rabbit, also a noble knight. Upon arrival, Pooh finds his friend sitting at a round table with 5 other guests.
One-by-one, Pooh asks each person at the table how many of his two neighbors are knights. Surprisingly, he gets the same answer from everybody! "Oh bother!" proclaims Pooh. "I still don't have enough information to figure out how many knights are at this table." "But it's my birthday," adds one of the guests. "Yes, it's his birthday!" agrees his neighbor.
Now Pooh can tell how many knights are at the table. Can you?

p2. Harry has an 8×8 board filled with the numbers 1 and -1, and the sum of all 64 numbers is 0. A magical cut of this board is a way of cutting it into two pieces so that the sum of the numbers in each piece is also 0. The pieces should not have any holes. Prove that Harry will always be able to find a magical cut of his board. (The picture shows an example of a proper cut.)

https://cdn.artofproblemsolving.com/attachments/4/b/98dec239cfc757e6f2996eef7876cbfd79d20
png

p3. Several girls participate in a tennis tournament in which each player plays each other player exactly once. At the end of the tournament, it turns out that each player has lost at least one of her games. Prove that it is possible to find three players A, B, and C such that A defeated B, B defeated C, and C defeated A.

p4. 120 bands are participating in this year's Northwest Grunge Rock Festival, and they have 119 fans in total. Each fan belongs to exactly one fan club. A fan club is called crowded if it has at least 15 members.
Every morning, all the members of one of the crowded fan clubs start arguing over who loves their favorite band the most. As a result of the fighting, each of them leaves the club to join another club, but no two of them join the same one.
Is it true that, no matter how the clubs are originally arranged, all these arguments will eventually stop?

p5. In Infinite City, the streets form a grid of squares extending infinitely in all directions. Bonnie and Clyde have just robbed the Infinite City Bank, located at the busiest intersection downtown. Bonnie sets off heading north on her bike, and, 30 seconds later, Clyde bikes after her in the same direction. They each bike at a constant speed of 1 block per minute. In order to throw off any authorities, each of them must turn either left or right at every intersection. If they continue biking in this manner, will they ever be able to meet?

<u>Round 2</u>

p6. In a certain herd of 33 cows, each cow weighs a whole number of pounds. Farmer Dan notices that if he removes any one of the cows from the herd, it is possible to split the remaining 32 cows into two groups of equal total weight, 16 cows in each group. Show that all 33 cows must have the same weight.

p7. Katniss is thinking of a positive integer less than 100: call it x. Peeta is allowed to pick any two positive integers N and M, both less than 100, and Katniss will give him the greatest common divisor of $x + M$ and N. Peeta can do this up to seven times, after which he must name Katniss' number x, or he will die. Can Peeta ensure his survival?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).