



individual oral math Olympiad by University of Washington , Grades 6-7 and 8-10

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by parmenides51

6-7 Round 1

p1. Goldilocks enters the home of the three bears – Papa Bear, Mama Bear, and Baby Bear. Each bear is wearing a different-colored shirt – red, green, or blue. All the bears look the same to Goldilocks, so she cannot otherwise tell them apart.

The bears in the red and blue shirts each make one true statement and one false statement.

The bear in the red shirt says: "I'm Blue's dad. I'm Green's daughter."

The bear in the blue shirt says: "Red and Green are of opposite gender. Red and Green are my parents."

Help Goldilocks find out which bear is wearing which shirt.

p2. The University of Washington is holding a talent competition. The competition has five contests: math, physics, chemistry, biology, and ballroom dancing. Any student can enter into any number of the contests but only once for each one. For example, a student may participate in math, biology, and ballroom.

It turned out that each student participated in an odd number of contests. Also, each contest had an odd number of participants. Was the total number of contestants odd or even?

p3. The 99 greatest scientists of Mars and Venus are seated evenly around a circular table. If any scientist sees two colleagues from her own planet sitting an equal number of seats to her left and right, she waves to them. For example, if you are from Mars and the scientists sitting two seats to your left and right are also from Mars, you will wave to them. Prove that at least one of the 99 scientists will be waving, no matter how they are seated around the table.

p4. One hundred boys participated in a tennis tournament in which every player played each other player exactly once and there were no ties. Prove that after the tournament, it is possible for the boys to line up for pizza so that each boy defeated the boy standing right behind him in line.

p5. To celebrate space exploration, the Science Fiction Museum is going to read Star Wars and Star Trek stories for 24 hours straight. A different story will be read each hour for a total of 12 Star Wars stories and 12 Star Trek stories. George and Gene want to listen to exactly 6 Star Wars and 6 Star Trek stories. Show that no matter how the readings are scheduled, the friends can find a block of 12 consecutive hours to listen to the stories together.

Round 2

p6. 2013 people attended Cinderella's ball. Some of the guests were friends with each other. At midnight, the guests started turning into mice. After the first minute, everyone who had no friends at the ball turned into a mouse. After the second minute, everyone who had exactly one friend among the remaining people turned into a mouse. After the third minute, everyone who had two human friends left in the room turned into a mouse, and so on. What is the maximal number of people that could have been left at the ball after 2013 minutes?

p7. Bill and Charlie are playing a game on an infinite strip of graph paper. On Bill's turn, he marks two empty squares of his choice (not necessarily adjacent) with crosses. Charlie, on his turn, can erase any number of crosses, as long as they are all adjacent to each other. Bill wants to create a line of 2013 crosses in a row. Can Charlie stop him?

PS. You should use hide for answers. Collected here (<https://artofproblemsolving.com/community/c5h2760506p24143309>).

8-10 Round 1

p1. Pirate Jim had 8 boxes with gun powder weighing 1, 2, 3, 4, 5, 6, 7, and 8 pounds (the weight is printed on top of every box). Pirate Bob hid a 1-pound gold bar in one of these boxes. Pirate Jim has a balance scale that he can use, but he cannot open any of the boxes. Help him find the box with the gold bar using two weighings on the balance scale.

p2. James Bond will spend one day at Dr. Evil's mansion to try to determine the answers to two questions:

- a) Is Dr. Evil at home?
- b) Does Dr. Evil have an army of ninjas?

The parlor in Dr. Evil's mansion has three windows. At noon, Mr. Bond will sneak into the parlor and use open or closed windows to signal his answers. When he enters the parlor, some windows may already be opened, and Mr. Bond will only have time to open or close one window (or leave them all as they are).

Help Mr. Bond and Moneypenny design a code that will tell Moneypenny the answers to both questions when she drives by later that night and looks at the windows. Note that Moneypenny will not have any way to know which window Mr. Bond opened or closed.

p3. Suppose that you have a triangle in which all three side lengths and all three heights are integers. Prove that if these six lengths are all different, there cannot be four prime numbers

among them.

p4. Fred and George have designed the Amazing Maze, a 5×5 grid of rooms, with Adorable Doors in each wall between rooms. If you pass through a door in one direction, you gain a gold coin. If you pass through the same door in the opposite direction, you lose a gold coin. The brothers designed the maze so that if you ever come back to the room in which you started, you will find that your money has not changed.

Ron entered the northwest corner of the maze with no money. After walking through the maze for a while, he had 8 shiny gold coins in his pocket, at which point he magically teleported himself out of the maze. Knowing this, can you determine whether you will gain or lose a coin when you leave the central room through the north door?

p5. Bill and Charlie are playing a game on an infinite strip of graph paper. On Bill's turn, he marks two empty squares of his choice (not necessarily adjacent) with crosses. Charlie, on his turn, can erase any number of crosses, as long as they are all adjacent to each other. Bill wants to create a line of 2013 crosses in a row. Can Charlie stop him?

Round 2

p6. 1000 non-zero numbers are written around a circle and every other number is underlined. It happens that each underlined number is equal to the sum of its two neighbors and that each non-underlined number is equal to the product of its two neighbors. What could the sum of all the numbers written on the circle be?

p7. A grasshopper is sitting at the edge of a circle of radius 3 inches. He can hop exactly 4 inches in any direction, as long as he stays within the circle. Which points inside the circle can the grasshopper reach if he can make as many jumps as he likes?

<https://cdn.artofproblemsolving.com/attachments/1/d/39b34b2b4afe607c1232f4ce9dec040a34b0c.png>

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