individual oral math Olympiad by University of Washington , Grades 5-7 and 8-10

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5-7   <u>Round 1</u>

p1. Three snails – Alice, Bobby, and Cindy – were racing down a road.
Whenever one snail passed another, it waved at the snail it passed.
During the race, Alice waved 3 times and was waved at twice.
Bobby waved 4 times and was waved at 3 times.
Cindy waved 5 times. How many times was she waved at?

p2. Sherlock and Mycroft are playing Battleship on a 4×4 grid. Mycroft hides a single 3×1 cruiser somewhere on the board. Sherlock can pick squares on the grid and fire upon them. What is the smallest number of shots Sherlock has to fire to guarantee at least one hit on the cruiser?

p3. Thirty girls – 13 of them in red dresses and 17 in blue dresses – were dancing in a circle, hand-in-hand. Afterwards, each girl was asked if the girl to her right was in a blue dress. Only the girls who had both neighbors in red dresses or both in blue dresses told the truth. How many girls could have answered "Yes"?

p4. Herman and Alex play a game on a 5×5 board. On his turn, a player can claim any open square as his territory. Once all the squares are claimed, the winner is the player whose territory has the longer border. Herman goes first. If both play their best, who will win, or will the game end in a draw?
https://cdn.artofproblemsolving.com/attachments/5/7/113d54f2217a39bac622899d3d3eb51ec34f1
png

p5. Is it possible to find 2014 distinct positive integers whose sum is divisible by each of them?

<u>Round 2</u>

p6. Hermione and Ron play a game that starts with 129 hats arranged in a circle. They take turns magically transforming the hats into animals. On each turn, a player picks a hat and chooses whether to change it into a badger or into a raven. A player loses if after his or her turn there are two animals of the same species right next to each other. Hermione goes first. Who loses?

p7. Three warring states control the corner provinces of the island whose map is shown below.
`https://cdn.artofproblemsolving.com/attachments/e/a/4e2f436be1dcd3f899aa34145356f8c66cda8`
`png`
As a result of war, each of the remaining 18 provinces was occupied by one of the states. None of the states was able to occupy any province on the coast opposite their corner. The states would like to sign a peace treaty. To do this, they each must send ambassadors to a place where three provinces, one controlled by each state, come together. Prove that they can always find such a place to meet.
For example, if the provinces are occupied as shown here, the squares mark possible meeting spots.
`https://cdn.artofproblemsolving.com/attachments/e/b/81de9187951822120fc26024c1c1fbe213873`
`png`

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

8-10.1 Sherlock and Mycroft are playing Battleship on a 4×4 grid. Mycroft hides a single 3×1 cruiser somewhere on the board. Sherlock can pick squares on the grid and fire upon them. What is the smallest number of shots Sherlock has to fire to guarantee at least one hit on the cruiser?

8-10.2 A complete set of the Encyclopedia of Mathematics has 10 volumes. There are ten mathematicians in Mathemagic Land, and each of them owns two volumes of the Encyclopedia. Together they own two complete sets. Show that there is a way for each mathematician to donate one book to the library such that the library receives a complete set.

8-10.3 There are 2014 airports in the faraway land of Artinia. Each pair of airports is connected by a nonstop flight in one or both directions. Show that there is some airport from which it is possible to reach every other airport in at most two flights.

8-10.4 Hermione and Ron play a game that starts with 129 hats arranged in a circle. They take turns magically transforming the hats into animals. On each turn, a player picks a hat and chooses whether to change it into a badger or into a raven.

A player loses if after his or her turn there are two animals of the same species right next to each other. Hermione goes first. Who loses?

8-10.5 An infinite number of lilypads grow in a line, numbered $\ldots, -2, -1, 0, 1, 2, \ldots$ Thumbelina and her pet frog start on one of the lilypads. She wants to make a sequence of jumps that will end on either pad 0 or pad 96. On each jump, Thumbelina tells her frog the distance (number of pads) to leap, but the frog chooses whether to jump left or right. From which starting pads can she always get to pad 0 or pad 96, regardless of her frog's decisions?

8-10.6 Homer goes on the 100-Donut Diet. A 100-Donut Diet Plan specifies how many of 100 total donuts Homer will eat each day. The diet requires that the number of donuts he eats does not increase from one day to the next. For example, one 5-day Donut Diet Plan is $40, 25, 25, 8, 2$.

Are there more 100-Donut Diet Plans with an odd number of days or plans where Homer eats an odd number of donuts on the first day?

8-10.7 If a is any number, $\lfloor a \rfloor$ is a rounded down to the nearest integer. For example, $\lfloor \pi \rfloor = 3$. Show that the sequence

$$\lfloor \tfrac{2^1}{17} \rfloor, \lfloor \tfrac{2^2}{17} \rfloor, \lfloor \tfrac{2^3}{17} \rfloor, \ldots$$

contains infinitely many odd numbers.