individual oral math Olympiad by University of Washington , Grades 6-7 and 8-10

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6-7 Round 1

p1. At a fortune-telling exam, 13 witches are sitting in a circle. To pass the exam, a witch must correctly predict, for everybody except herself and her two neighbors, whether they will pass or fail. Each witch predicts that each of the 10 witches she is asked about will fail. How many witches could pass?

p2. Out of 152 coins, 7 are counterfeit. All counterfeit coins have the same weight, and all real coins have the same weight, but counterfeit coins are lighter than real coins. How can you find 19 real coins if you are allowed to use a balance scale three times?

p3. The digits of a number N increase from left to right. What could the sum of the digits of $9 \times N$ be?

p4. The sides and diagonals of a pentagon are colored either blue or red. You can choose three vertices and flip the colors of all three lines that join them. Can every possible coloring be turned all blue by a sequence of such moves?
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p5. You have 100 pancakes, one with a single blueberry, one with two blueberries, one with three blueberries, and so on. The pancakes are stacked in a random order. Count the number of blueberries in the top pancake and call that number N. Pick up the stack of the top N pancakes and flip it upside down. Prove that if you repeat this counting-and-flipping process, the pancake with one blueberry will eventually end up at the top of the stack.

Round 2

p6. A circus owner will arrange 100 fleas on a long string of beads, each flea on her own bead. Once arranged, the fleas start jumping using the following rules. Every second, each flea chooses the closest bead occupied by one or more of the other fleas, and then all fleas jump simultaneously to their chosen beads. If there are two places where a flea could jump, she jumps to the right. At the start, the circus owner arranged the fleas so that, after some time, they all gather on just two beads. What is the shortest amount of time it could take for this to happen?

p7. The faraway land of Noetheria has 2016 cities. There is a nonstop flight between every pair of cities. The price of a nonstop ticket is the same in both directions, but flights between different pairs of cities have different prices. Prove that you can plan a route of 2015 consecutive flights so that each flight is cheaper than the previous one. It is permissible to visit the same city several times along the way.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

8-10 Round 1

p1. Alice and Bob compiled a list of movies that exactly one of them saw, then Cindy and Dale did the same. To their surprise, these two lists were identical. Prove that if Alice and Cindy list all movies that exactly one of them saw, this list will be identical to the one for Bob and Dale.

p2. Several whole rounds of cheese were stored in a pantry. One night some rats sneaked in and consumed 10 of the rounds, each rat eating an equal portion. Some were satisfied, but 7 greedy rats returned the next night to finish the remaining rounds. Their portions on the second night happened to be half as large as on the first night. How many rounds of cheese were initially in the pantry?

p3. You have 100 pancakes, one with a single blueberry, one with two blueberries, one with three blueberries, and so on. The pancakes are stacked in a random order.
Count the number of blueberries in the top pancake, and call that number N. Pick up the stack of the top N pancakes, and flip it upside down. Prove that if you repeat this counting-and-flipping process, the pancake with one blueberry will eventually end up at the top of the stack.

p4. There are two lemonade stands along the 4-mile-long circular road that surrounds Sour Lake. 100 children live in houses along the road. Every day, each child buys a glass of lemonade from the stand that is closest to her house, as long as she does not have to walk more than one mile along the road to get there.
A stand's advantage is the difference between the number of glasses it sells and the number of glasses its competitor sells. The stands are positioned such that neither stand can increase its advantage by moving to a new location, if the other stand stays still. What is the maximum number of kids who can't buy lemonade (because both stands are too far away)?

p5. Merlin uses several spells to move around his 64-room castle. When Merlin casts a spell in a room, he ends up in a different room of the castle. Where he ends up only depends on the room where he cast the spell and which spell he cast. The castle has the following magic property: if a sequence of spells brings Merlin from some room A back to room A, then from any other room B in the castle, that same sequence brings Merlin back to room B. Prove that there are two different rooms X and Y and a sequence of spells that both takes Merlin from X to Y and from Y to X.

Round 2

p6. Captains Hook, Line, and Sinker are deciding where to hide their treasure. It is currently buried at the X in the map below, near the lairs of the three pirates. Each pirate would prefer that the treasure be located as close to his own lair as possible. You are allowed to propose a new location for the treasure to the pirates. If at least two out of the three pirates prefer the new location (because it moves closer to their own lairs), then the treasure will be moved there. Assuming the pirates' lairs form an acute triangle, is it always possible to propose a sequence of new locations so that the treasure eventually ends up in your backyard (wherever that is)?
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p7. Homer went on a Donut Diet for the month of May (31 days). He ate at least one donut every day of the month. However, over any stretch of 7 consecutive days, he did not eat more than 13 donuts. Prove that there was some stretch of consecutive days over which Homer ate exactly 30 donuts.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).