

**individual oral math Olympiad by University of Washington , Grades 6-7 and 8-10**

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by parmenides51

**6-7** Round 1

**p1.** Ten children arrive at a birthday party and leave their shoes by the door. All the children have different shoe sizes. Later, as they leave one at a time, each child randomly grabs a pair of shoes their size or larger. After some kids have left, all of the remaining shoes are too small for any of the remaining children. What is the greatest number of shoes that might remain by the door?

**p2.** Turans, the king of Saturn, invented a new language for his people. The alphabet has only 6 letters: A, N, R, S, T, U; however, the alphabetic order is different than in English. A word is any sequence of 6 different letters. In the dictionary for this language, the first word is SATURN. Which word follows immediately after TURANS?

**p3.** Benji chooses five integers. For each pair of these numbers, he writes down the pair's sum. Can all ten sums end with different digits?

**p4.** Nine dwarves live in a house with nine rooms arranged in a  $3 \times 3$  square. On Monday morning, each dwarf rubs noses with the dwarves in the adjacent rooms that share a wall. On Monday night, all the dwarves switch rooms. On Tuesday morning, they again rub noses with their adjacent neighbors. On Tuesday night, they move again. On Wednesday morning, they rub noses for the last time. Show that there are still two dwarves who haven't rubbed noses with one another.

**p5.** Anna and Bobby take turns placing rooks in any empty square of a pyramid-shaped board with 100 rows and 200 columns. If a player places a rook in a square that can be attacked by a previously placed rook, he or she loses. Anna goes first. Can Bobby win no matter how well Anna plays?

<https://cdn.artofproblemsolving.com/attachments/7/5/b253b655b6740b1e1310037da07a0df4dc991.png>

Round 2

**p6.** Some boys and girls, all of different ages, had a snowball fight. Each girl threw one snowball at every kid who was older than her. Each boy threw one snowball at every kid who was younger than him. Three friends were hit by the same number of snowballs, and everyone else took fewer

hits than they did. Prove that at least one of the three is a girl.

**p7.** Last year, jugglers from around the world travelled to Jakarta to participate in the Jubilant Juggling Jamboree. The festival lasted 32 days, with six solo performances scheduled each day. The organizers noticed that for any two days, there was exactly one juggler scheduled to perform on both days. No juggler performed more than once on a single day. Prove there was a juggler who performed every day.

PS. You should use hide for answers. Collected here (<https://artofproblemsolving.com/community/c5h2760506p24143309>).

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### 8-10 Round 1

**p1.** The Queen of Bees invented a new language for her hive. The alphabet has only 6 letters: A, C, E, N, R, T; however, the alphabetic order is different than in English. A word is any sequence of 6 different letters. In the dictionary for this language, the word TRANCE immediately follows NECTAR. What is the last word in the dictionary?

**p2.** Is it possible to solve the equation  $\frac{1}{x} = \frac{1}{y} + \frac{1}{z}$  with  $x, y, z$  integers (positive or negative) such that one of the numbers  $x, y, z$  has one digit, another has two digits, and the remaining one has three digits?

**p3.** The 10,000 dots in a  $100 \times 100$  square grid are all colored blue. Rekha can paint some of them red, but there must always be a blue dot on the line segment between any two red dots. What is the largest number of dots she can color red? The picture shows a possible coloring for a  $5 \times 7$  grid.

<https://cdn.artofproblemsolving.com/attachments/0/6/795f5ab879938ed2a4c8844092b873fb8589f.jpg>

**p4.** Six flies rest on a table. You have a swatter with a checkerboard pattern, much larger than the table. Show that there is always a way to position and orient the swatter to kill at least five of the flies. Each fly is much smaller than a swatter square and is killed if any portion of a black square hits any part of the fly.

**p5.** Maryam writes all the numbers  $1 - 81$  in the cells of a  $9 \times 9$  table. Tian calculates the product of the numbers in each of the nine rows, and Olga calculates the product of the numbers in every column. Could Tian's and Olga's lists of nine products be identical?

### Round 2

**p6.** A set of points in the plane is epic if, for every way of coloring the points red or blue, it is possible to draw two lines such that each blue point is on a line, but none of the red points are. The figure shows a particular set of 4 points and demonstrates that it is epic. What is the maximum possible size of an epic set?

<https://cdn.artofproblemsolving.com/attachments/e/f/44fd1679c520bdc55c78603190409222d0b72.jpg>

**p7.** Froggy Chess is a game played on a pond with lily pads. First Judit places a frog on a pad of her choice, then Magnus places a frog on a different pad of his choice. After that, they alternate turns, with Judit moving first. Each player, on his or her turn, selects either of the two frogs and another lily pad where that frog must jump. The jump must reduce the distance between the frogs (all distances between the lily pads are different), but both frogs cannot end up on the same lily pad. Whoever cannot make a move loses. The picture below shows the jumps permitted in a particular situation.

Who wins the game if there are 2017 lily pads?

<https://cdn.artofproblemsolving.com/attachments/a/9/1a26e046a2a614a663f9d317363aac6165468.jpg>

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