individual oral math Olympiad by University of Washington , Grades 6-7 and 8-10

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by parmenides51

6-7 Round 1

p1. Alice and Bob played 25 games of rock-paper-scissors. Alice played rock 12 times, scissors 6 times, and paper 7 times. Bob played rock 13 times, scissors 9 times, and paper 3 times. If there were no ties, who won the most games?
(Remember, in each game each player picks one of rock, paper, or scissors. Rock beats scissors, scissors beat paper, and paper beats rock. If they choose the same object, the result is a tie.)

p2. On the planet Vulcan there are eight big volcanoes and six small volcanoes. Big volcanoes erupt every three years and small volcanoes erupt every two years. In the past five years, there were 30 eruptions. How many volcanoes could erupt this year?

p3. A tangle is a sequence of digits constructed by picking a number $N \ge 0$ and writing the integers from 0 to N in some order, with no spaces. For example, 010123459876 is a tangle with $N = 10$. A palindromic sequence reads the same forward or backward, such as 878 or 6226. The shortest palindromic tangle is 0. How long is the second-shortest palindromic tangle?

p4. Balls numbered 1 to N have been randomly arranged in a long input tube that feeds into the upper left square of an 8×8 board. An empty exit tube leads out of the lower right square of the board. Your goal is to arrange the balls in order from 1 to N in the exit tube. As a move, you may
1. move the next ball in line from the input tube into the upper left square of the board,
2. move a ball already on the board to an adjacent square to its right or below, or
3. move a ball from the lower right square into the exit tube.
No square may ever hold more than one ball. What is the largest number N for which you can achieve your goal, no matter how the balls are initially arranged? You can see the order of the balls in the input tube before you start.
https://cdn.artofproblemsolving.com/attachments/1/8/bbce92750b01052db82d58b96584a36fb5ca5
png

p5. A 2018×2018 board is covered by non-overlapping 2×1 dominoes, with each domino covering two squares of the board. From a given square, a robot takes one step to the other square of the domino it is on and then takes one more step in the same direction. Could the robot continue moving this way forever without falling off the board?
https://cdn.artofproblemsolving.com/attachments/9/c/da86ca4ff0300eca8e625dff891ed1769d44a

png

Round 2

p6. Seventeen teams participated in a soccer tournament where a win is worth 1 point, a tie is worth 0 points, and a loss is worth -1 point. Each team played each other team exactly once. At least $\frac{3}{4}$ of all games ended in a tie. Show that there must be two teams with the same number of points at the end of the tournament.

p7. The city of Old Haven is known for having a large number of secret societies. Any person may be a member of multiple societies. A secret society is called influential if its membership includes at least half the population of Old Haven. Today, there are 2018 influential secret societies. Show that it is possible to form a council of at most 11 people such that each influential secret society has at least one member on the council.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

8-10 Round 1

p1. Five children, Aisha, Baesha, Cosha, Dasha, and Erisha, competed in running, jumping, and throwing. In each event, first place was won by someone from Renton, second place by someone from Seattle, and third place by someone from Tacoma. Aisha was last in running, Cosha was last in jumping, and Erisha was last in throwing. Could Baesha and Dasha be from the same city?

p2. Fifty-five Brits and Italians met in a coffee shop, and each of them ordered either coffee or tea. Brits tell the truth when they drink tea and lie when they drink coffee; Italians do it the other way around. A reporter ran a quick survey:
Forty-four people answered ”yes” to the question, ”Are you drinking coffee?”
Thirty-three people answered ”yes” to the question, ”Are you Italian?”
Twenty-two people agreed with the statement, ”It is raining outside.”
How many Brits in the coffee shop are drinking tea?

p3. Doctor Strange is lost in a strange house with a large number of identical rooms, connected to each other in a loop. Each room has a light and a switch that could be turned on and off. The lights might initially be on in some rooms and off in others. How can Dr. Strange determine the number of rooms in the house if he is only allowed to switch lights on and off?

p4. Fifty street artists are scheduled to give solo shows with three consecutive acts: juggling, drumming, and gymnastics, in that order. Each artist will spend equal time on each of the three activities, but the lengths may be different for different artists. At least one artist will be drumming at every moment from dawn to dusk. A new law was just passed that says two artists may not drum at the same time. Show that it is possible to cancel some of the artists' complete shows, without rescheduling the rest, so that at least one show is going on at every moment from dawn to dusk, and the schedule complies with the new law.

p5. Alice and Bob split the numbers from 1 to 12 into two piles with six numbers in each pile. Alice lists the numbers in the first pile in increasing order as $a_1 < a_2 < a_3 < a_4 < a_5 < a_6$ and Bob lists the numbers in the second pile in decreasing order $b_1 > b_1 > b_3 > b_4 > b_5 > b_6$. Show that no matter how they split the numbers,

$$|a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3| + |a_4 - b_4| + |a_5 - b_5| + |a_6 - b_6| = 36.$$

Round 2

p6. The Martian alphabet has ? letters. Marvin writes down a word and notices that within every sub-word (a contiguous stretch of letters) at least one letter occurs an odd number of times. What is the length of the longest possible word he could have written?

p7. For a long space journey, two astronauts with compatible personalities are to be selected from 24 candidates. To find a good fit, each candidate was asked 24 questions that required a simple yes or no answer. Two astronauts are compatible if exactly 12 of their answers matched (that is, both answered yes or both answered no). Miraculously, every pair of these 24 astronauts was compatible! Show that there were exactly 12 astronauts whose answer to the question "Can you repair a flux capacitor?" was exactly the same as their answer to the question "Are you afraid of heights?" (that is, yes to both or no to both).

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