**individual oral math Olympiad by University of Washington , Grades 6-7 and 8-10**
www.artofproblemsolving.com/community/c3168296
by parmenides51

**6-7** Round 1

**p1.** Three two-digit numbers are written on a board. One starts with $5$, another with $6$, and the last one with $7$. Annie added the first and the second numbers; Benny added the second and the third numbers; Denny added the third and the first numbers. Could it be that one of these sums is equal to $148$, and the two other sums are three-digit numbers that both start with $12$?

**p2.** Three rocks, three seashells, and one pearl are placed in identical boxes on a circular plate in the order shown. The lids of the boxes are then closed, and the plate is secretly rotated. You can open one box at a time. What is the smallest number of boxes you need to open to know where the pearl is, no matter how the plate was rotated?
https://cdn.artofproblemsolving.com/attachments/0/2/6bb3a2a27f417a84ab9a64100b90b8768f797
png

**p3.** Two detectives, Holmes and Watson, are hunting the thief Raffles in a library, which has the floorplan exactly as shown in the diagram. Holmes and Watson start from the center room marked $D$. Show that no matter where Raffles is or how he moves, Holmes and Watson can find him. Holmes and Watson do not need to stay together. A detective sees Raffles only if they are in the same room. A detective cannot stand in a doorway to see two rooms at the same time.
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png

**p4.** A museum has a $4 \times 4$ grid of rooms. Every two rooms that share a wall are connected by a door. Each room contains some paintings. The total number of paintings along any path of $7$ rooms from the lower left to the upper right room is always the same. Furthermore, the total number of paintings along any path of $7$ rooms from the lower right to the upper left room is always the same. The guide states that the museum has exactly $500$ paintings. Show that the guide is mistaken.
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png

**p5.** The numbers $1$–$14$ are placed around a circle in some order. You can swap two neighbors if they differ by more than $1$. Is it always possible to rearrange the numbers using swaps so they are ordered clockwise from $1$ to $14$?

 

Round 2

**p6.** A triangulation of a regular polygon is a way of drawing line segments between its vertices so that no two segments cross, and the interior of the polygon is divided into triangles. A flip move erases a line segment between two triangles, creating a quadrilateral, and replaces it with the opposite diagonal through that quadrilateral. This results in a new triangulation.
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Given any two triangulations of a polygon, is it always possible to find a sequence of flip moves that transforms the first one into the second one?
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png

**p7.** Is it possible to place the numbers from $1$ to $121$ in an $11\times 11$ table so that numbers that differ by $1$ are in horizontally or vertically adjacent cells and all the perfect squares $(1, 4, 9, ..., 121)$ are in one column?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

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**8-10** Round 1

**p1.** The alphabet of the Aau-Bau language consists of two letters: A and B. Two words have the same meaning if one of them can be constructed from the other by replacing any AA with A, replacing any BB with B, or by replacing any ABA with BAB. For example, the word AABA means the same thing as ABA, and AABA also means the same thing as ABAB. In this language, is it possible to name all seven days of the week?

**p2.** A museum has a $4 \times 4$ grid of rooms. Every two rooms that share a wall are connected by a door. Each room contains some paintings. The total number of paintings along any path of $7$ rooms from the lower left to the upper right room is always the same. Furthermore, the total number of paintings along any path of $7$ rooms from the lower right to the upper left room is always the same. The guide states that the museum has exactly $500$ paintings. Show that the guide is mistaken.
https://cdn.artofproblemsolving.com/attachments/7/6/0fd93a0deaa71a5bb1599d2488f8b4eac5d0e
jpg

**p3.** A playground has a swing-set with exactly three swings. When 3rd and 4th graders from Dr.

Anna's math class play during recess, she has a rule that if a $3^{rd}$ grader is in the middle swing there must be $4^{th}$ graders on that person's left and right. And if there is a $4^{th}$ grader in the middle, there must be $3^{rd}$ graders on that person's left and right. Dr. Anna calculates that there are $350$ different ways her students can arrange themselves on the three swings with no empty seats. How many students are in her class?
https://cdn.artofproblemsolving.com/attachments/5/9/4c402d143646582376d09ebbe54816b879931 jpg

**p4.** The archipelago Artinagos has $19$ islands. Each island has toll bridges to at least $3$ other islands. An unsuspecting driver used a bad mapping app to plan a route from North Noether Island to South Noether Island, which involved crossing $12$ bridges. Show that there must be a route with fewer bridges.
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**p5.** Is it possible to place the numbers from $1$ to $121$ in an $11\times 11$ table so that numbers that differ by $1$ are in horizontally or vertically adjacent cells and all the perfect squares $(1, 4, 9, ..., 121)$ are in one column?

Round 2

**p6.** Hungry and Sneaky have opened a rectangular box of chocolates and are going to take turns eating them. The chocolates are arranged in a $2m \times 2n$ grid. Hungry can take any two chocolates that are side-by-side, but Sneaky can take only one at a time. If there are no more chocolates located side-by-side, all remaining chocolates go to Sneaky. Hungry goes first. Each player wants to eat as many chocolates as possible. What is the maximum number of chocolates Sneaky can get, no matter how Hungry picks his?
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**p7.** There is a thief hiding in the sultan's palace. The palace contains $2019$ rooms connected by doors. One can walk from any room to any other room, possibly through other rooms, and there is only one way to do this. That is, one cannot walk in a loop in the palace. To catch the thief, a guard must be in the same room as the thief at the same time. Prove that $11$ guards can always find and catch the thief, no matter how the thief moves around during the search.
https://cdn.artofproblemsolving.com/attachments/a/b/9728ac271e84c4954935553c4d58b3ff4b194 jpg

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

 