www.artofproblemsolving.com/community/c3168315
by parmenides51

– Team Round

– **p1.** Suppose 5 bales of hay are weighted two at a time in all possible ways. The weights obtained are 110, 112, 113, 114, 115, 116, 117, 118, 120, 121. What is the difference between the heaviest and the lightest bale?

p2. Paul and Paula are playing a game with dice. Each have an 8-sided die, and they roll at the same time. If the number is the same they continue rolling; otherwise the one who rolled a higher number wins. What is the probability that the game lasts at most 3 rounds?

p3. Find the unique positive integer n such that $\frac{n^3+5}{n^2-1}$ is an integer.

p4. How many numbers have 6 digits, some four of which are $2, 0, 1, 4$ (not necessarily consecutive or in that order) and have the sum of their digits equal to 9?

p5. The Duke School has N students, where N is at most 500. Every year the school has three sports competitions: one in basketball, one in volleyball, and one in soccer. Students may participate in all three competitions. A basketball team has 5 spots, a volleyball team has 6 spots, and a soccer team has 11 spots on the team. All students are encouraged to play, but 16 people choose not to play basketball, 9 choose not to play volleyball and 5 choose not to play soccer. Miraculously, other than that all of the students who wanted to play could be divided evenly into teams of the appropriate size. How many players are there in the school?

p6. Let $\{a_n\}_{n\ge 1}$ be a sequence of real numbers such that $a_1 = 0$ and $a_{n+1} = \frac{a_n-\sqrt3}{\sqrt3 a_n+1}$. Find $a_1 + a_2 + .. + a_{2014}$.

p7. A soldier is fighting a three-headed dragon. At any minute, the soldier swings her sword, at which point there are three outcomes: either the soldier misses and the dragon grows a new head, the soldier chops off one head that instantaneously regrows, or the soldier chops off two heads and none grow back. If the dragon has at least two heads, the soldier is equally likely to miss or chop off two heads. The dragon dies when it has no heads left, and it overpowers the soldier if it has at least five heads. What is the probability that the soldier wins

p8. A rook moves alternating horizontally and vertically on an infinite chessboard. The rook moves one square horizontally (in either direction) at the first move, two squares vertically at the second, three horizontally at the third and so on. Let S be the set of integers n with the property that there exists a series of moves such that after the n-th move the rock is back where it started. Find the number of elements in the set $S \cap \{1, 2, ..., 2014\}$.

p9. Find the largest integer n such that the number of positive integer divisors of n (including 1 and n) is at least $\sqrt{n}$.

p10. Suppose that x, y are irrational numbers such that xy, $x^2 + y$, $y^2 + x$ are rational numbers. Find $x + y$.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

– Individual Round

– **p1.** p, q, r are prime numbers such that $p^q + 1 = r$. Find $p + q + r$.

p2. 2014 apples are distributed among a number of children such that each child gets a different number of apples. Every child gets at least one apple. What is the maximum possible number of children who receive apples?

p3. Cathy has a jar containing jelly beans. At the beginning of each minute he takes jelly beans out of the jar. At the n-th minute, if n is odd, he takes out 5 jellies. If n is even he takes out n jellies. After the 46th minute there are only 4 jellies in the jar. How many jellies were in the jar in the beginning?

p4. David is traveling to Budapest from Paris without a cellphone and he needs to use a public payphone. He only has two coins with him. There are three pay-phones - one that never works, one that works half of the time, and one that always works. The first phone that David tries does not work. Assuming that he does not use the same phone again, what is the probability that the second phone that he uses will work?

p5. Let a, b, c, d be positive real numbers such that

$$a^2 + b^2 = 1$$

$$c^2 + d^2 = 1;$$

$$ad - bc = \frac{1}{7}$$

Find $ac + bd$.

p6. Three circles C_A, C_B, C_C of radius 1 are centered at points A, B, C such that A lies on C_B and C_C, B lies on C_C and C_A, and C lies on C_A and C_B. Find the area of the region where C_A, C_B, and C_C all overlap.

p7. Two distinct numbers a and b are randomly and uniformly chosen from the set $\{3, 8, 16, 18, 24\}$. What is the probability that there exist integers c and d such that $ac + bd = 6$?

p8. Let S be the set of integers $1 \le N \le 2^{20}$ such that $N = 2^i + 2^j$ where i, j are distinct integers. What is the probability that a randomly chosen element of S will be divisible by 9?

p9. Given a two-pan balance, what is the minimum number of weights you must have to weigh any object that weighs an integer number of kilograms not exceeding 100 kilograms?

p10. Alex, Michael and Will write 2-digit perfect squares A, M, W on the board. They notice that the 6-digit number $10000A + 100M + W$ is also a perfect square. Given that $A < W$, find the square root of the 6-digit number.

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– Tiebreaker Round

– **p1.** A light beam shines from the origin into the unit square at an angle of θ to one of the sides such that $\tan \theta = \frac{13}{17}$. The light beam is reflected by the sides of the square. How many times does the light beam hit a side of the square before hitting a vertex of the square?
`https://cdn.artofproblemsolving.com/attachments/5/7/1db0aad33ed9bf82bee3303c7fbbe0b7c2574`
`png`

p2. Alex is given points $A_1, A_2, ..., A_{150}$ in the plane such that no three are collinear and A_1, A_2, ..., A_{100} are the vertices of a convex polygon P containing A_{101}, A_{102}, ..., A_{150} in its interior. He proceeds to draw edges A_iA_j such that no two edges intersect (except possibly at their end-points), eventually dividing P up into triangles. How many triangles are there?
`https://cdn.artofproblemsolving.com/attachments/d/5/12c757077e87809837d16128b018895a8bcc9`
`png`

p3. The polynomial P(x) has the property that $P(1)$, $P(2)$, $P(3)$, $P(4)$, and $P(5)$ are equal to $1, 2, 3,$ 4,5 in some order. How many possibilities are there for the polynomial P, given that the degree of P is strictly less than 4?

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