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– Team Round

– **p1.** Steven has just learned about polynomials and he is struggling with the following problem: expand $(1-2x)^7$ as $a_0 + a_1x + ... + a_7x^7$. Help Steven solve this problem by telling him what $a_1 + a_2 + ... + a_7$ is.

p2. Each element of the set $2, 3, 4, ..., 100$ is colored. A number has the same color as any divisor of it. What is the maximum number of colors?

p3. Fuchsia is selecting 24 balls out of 3 boxes. One box contains blue balls, one red balls and one yellow balls. They each have a hundred balls. It is required that she takes at least one ball from each box and that the numbers of balls selected from each box are distinct. In how many ways can she select the 24 balls?

p4. Find the perfect square that can be written in the form $\overline{abcd} - \overline{dcba}$ where a, b, c, d are non zero digits and $b < c$. $\overline{abcd}$ is the number in base 10 with digits a, b, c, d written in this order.

p5. Steven has 100 boxes labeled from 1 to 100. Every box contains at most 10 balls. The number of balls in boxes labeled with consecutive numbers differ by 1. The boxes labeled $1, 4, 7, 10, ..., 100$ have a total of 301 balls. What is the maximum number of balls Steven can have?

p6. In acute $\vartriangle ABC$, $AB = 4$. Let D be the point on BC such that $\angle BAD = \angle CAD$. Let AD intersect the circumcircle of $\vartriangle ABC$ at X. Let Γ be the circle through D and X that is tangent to AB at P. If $AP = 6$, compute AC.

p7. Consider a 15×15 square decomposed into unit squares. Consider a coloring of the vertices of the unit squares into two colors, red and blue such that there are 133 red vertices. Out of these 133, two vertices are vertices of the big square and 32 of them are located on the sides of the big square. The sides of the unit squares are colored into three colors. If both endpoints of a side are colored red then the side is colored red. If both endpoints of a side are colored blue then the side is colored blue. Otherwise the side is colored green. If we have 196 green sides, how many blue sides do we have?

p8. Carl has 10 piles of rocks, each pile with a different number of rocks. He notices that he can redistribute the rocks in any pile to the other 9 piles to make the other 9 piles have the same number of rocks. What is the minimum number of rocks in the biggest pile?

p9. Suppose that Tony picks a random integer between 1 and 6 inclusive such that the probability that he picks a number is directly proportional to the the number itself. Danny picks a number between 1 and 7 inclusive using the same rule as Tony. What is the probability that Tony's number is greater than Danny's number?

p10. Mike wrote on the board the numbers $1, 2, ..., n$. At every step, he chooses two of these numbers, deletes them and replaces them with the least prime factor of their sum. He does this until he is left with the number 101 on the board. What is the minimum value of n for which this is possible?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

– Individual Round

– **p1.** Trung has 2 bells. One bell rings 6 times per hour and the other bell rings 10 times per hour. At the start of the hour both bells ring. After how much time will the bells ring again at the same time? Express your answer in hours.

p2. In a soccer tournament there are n teams participating. Each team plays every other team once. The matches can end in a win for one team or in a draw. If the match ends with a win, the winner gets 3 points and the loser gets 0. If the match ends in a draw, each team gets 1 point. At the end of the tournament the total number of points of all the teams is 21. Let p be the number of points of the team in the first place. Find $n + p$.

p3. What is the largest 3 digit number $\overline{abc}$ such that $b \cdot \overline{ac} = c \cdot \overline{ab} + 50$?

p4. Let s(n) be the number of quadruplets (x, y, z, t) of positive integers with the property that $n = x + y + z + t$. Find the smallest n such that $s(n) > 2014$.

p5. Consider a decomposition of a 10×10 chessboard into p disjoint rectangles such that each rectangle contains an integral number of squares and each rectangle contains an equal number of white squares as black squares. Furthermore, each rectangle has different number of squares inside. What is the maximum of p?

p6. If two points are selected at random from a straight line segment of length π, what is the probability that the distance between them is at least $\pi - 1$?

p7. Find the length n of the longest possible geometric progression $a_1, a_2, .., , a_n$ such that the a_i are distinct positive integers between 100 and 2014 inclusive.

p8. Feng is standing in front of a 100 story building with two identical crystal balls. A crystal ball will break if dropped from a certain floor m of the building or higher, but it will not break if it is dropped from a floor lower than m. What is the minimum number of times Feng needs to drop a ball in order to guarantee he determined m by the time all the crystal balls break?

p9. Let A and B be disjoint subsets of $\{1, 2, ..., 10\}$ such that the product of the elements of A is equal to the sum of the elements in B. Find how many such A and B exist.

p10. During the semester, the students in a math class are divided into groups of four such that every two groups have exactly 2 students in common and no two students are in all the groups together. Find the maximum number of such groups.

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