www.artofproblemsolving.com/community/c3168322
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– Team Round

– **p1.** What is the maximum number of T-shaped polyominos (shown below) that we can put into a 6×6 grid without any overlaps. The blocks can be rotated.
https://cdn.artofproblemsolving.com/attachments/7/6/468fd9b81e9115a4a98e4cbf6dedf47ce8349
png

p2. In triangle $\triangle ABC$, $\angle A = 30^o$. D is a point on AB such that $CD \perp AB$. E is a point on AC such that $BE \perp AC$. What is the value of $\frac{DE}{BC}$?

p3. Given that f(x) is a polynomial such that $2f(x) + f(1 - x) = x^2$. Find the sum of squares of the coefficients of $f(x)$.

p4. For each positive integer n, there exists a unique positive integer an such that $a_n^2 \le n < (a_n + 1)^2$. Given that $n = 15m^2$, where m is a positive integer greater than 1. Find the minimum possible value of $n - a_n^2$.

p5. What are the last two digits of $\lfloor (\sqrt5 + 2)^{2016} \rfloor$?

Note $\lfloor x \rfloor$ is the largest integer less or equal to x.

p6. Let f be a function that satisfies $f(2^a3^b)) = 3a + 5b$. What is the largest value of f over all numbers of the form $n = 2^a3^b$ where $n \le 10000$ and a, b are nonnegative integers.

p7. Find a multiple of 21 such that it has six more divisors of the form $4m+1$ than divisors of the form $4n + 3$ where m, n are integers. You can keep the number in its prime factorization form.

p8. Find

$$\sum_{i=0}^{100} \lfloor i^{3/2} \rfloor + \sum_{j=0}^{1000} \lfloor j^{2/3} \rfloor$$

where $\lfloor x \rfloor$ is the largest integer less or equal to x.

p9. Let A, B be two randomly chosen subsets of $\{1, 2, ...10\}$. What is the probability that one of the two subsets contains the other?

p10. We want to pick 5-person teams from a total of m people such that:
1. Any two teams must share exactly one member.
2. For every pair of people, there is a team in which they are teammates.
How many teams are there?
(Hint: m is determined by these conditions).

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

– Tiebreaker Round

– **p1.** How many ordered triples of integers (a, b, c) where $1 \le a, b, c \le 10$ are such that for every natural number, the equation $(a + n)x^2 + (b + 2n)x + c + n = 0$ has at least one real root?

p2. Find the smallest integer n such that we can cut a $n \times n$ grid into 5 rectangles with distinct side lengths in $\{1, 2, 3..., 10\}$. Every value is used exactly once.

p3. A plane is flying at constant altitude along a circle of radius 12 miles with center at a point A.The speed of the aircraft is v. At some moment in time, a missile is fired at the aircraft from the point A, which has speed v and is guided so that its velocity vector always points towards the aircraft. How far does the missile travel before colliding with the aircraft?

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– Individual Round

– **p1.** Trung took five tests this semester. For his first three tests, his average was 60, and for the fourth test he earned a 50. What must he have earned on his fifth test if his final average for all five tests was exactly 60?

p2. Find the number of pairs of integers (a, b) such that $20a + 16b = 2016 - ab$.

p3. Let $f : N \to N$ be a strictly increasing function with $f(1) = 2016$ and $f(2t) = f(t) + t$ for all $t \in N$. Find $f(2016)$.

p4. Circles of radius 7, 7, 18, and r are mutually externally tangent, where $r = \frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

p5. A point is chosen at random from within the circumcircle of a triangle with angles 45^o, 75^o, 60^o. What is the probability that the point is closer to the vertex with an angle of 45^o than either of the two other vertices?

p6. Find the largest positive integer a less than 100 such that for some positive integer b, $a - b$ is a prime number and ab is a perfect square.

p7. There is a set of 6 parallel lines and another set of six parallel lines, where these two sets of lines are not parallel with each other. If Blythe adds 6 more lines, not necessarily parallel with each other, find the maximum number of triangles that could be made.

p8. Triangle ABC has sides $AB = 5$, $AC = 4$, and $BC = 3$. Let O be any arbitrary point inside ABC, and $D \in BC$, $E \in AC$, $F \in AB$, such that $OD \perp BC$, $OE \perp AC$, $OF \perp AB$. Find the minimum value of $OD^2 + OE^2 + OF^2$.

p9. Find the root with the largest real part to $x^4 - 3x^3 + 3x + 1 = 0$ over the complex numbers.

p10. Tony has a board with 2 rows and 4 columns. Tony will use 8 numbers from 1 to 8 to fill in this board, each number in exactly one entry. Let array $(a_1, ..., a_4)$ be the first row of the board and array $(b_1, ..., b_4)$ be the second row of the board. Let $F = \sum_{i=1}^{4} |a_i - b_i|$, calculate the average value of F across all possible ways to fill in.

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