Kurschak Competition 2022

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by MathHorse

1 A square has been divided into 2022 rectangles with no two of them having a common interior point. What is the maximal number of distinct lines that can be determined by the sides of these rectangles?

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2 Let p and q be prime numbers of the form $4k+3$. Suppose that there exist integers x and y such that $x^2-pqy^2=1$. Prove that there exist positive integers a and b such that $|pa^2-qb^2|=1$.

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3 Let $a_{i,j}\ \ (\forall\ \ 1\le i\le n, 1\le j\le n)$ be n^2 real numbers such that $a_{i,j}+a_{j,i}=0\ \ \forall i,j$ (in particular, $a_{i,i}=0\ \ \forall i$). Prove that

$$\frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{n}a_{i,j}\right)^2\le\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{i,j}^2.$$
