Kurschak Competition 1980

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1 The points of space are coloured with five colours, with all colours being used. Prove that some plane contains four points of different colours.

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2 Let $n > 1$ be an odd integer. Prove that a necessary and sufficient condition for the existence of positive integers x and y satisfying

$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y}$$

is that n has a prime divisor of the form $4k - 1$.

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3 In a certain country there are two tennis clubs consisting of 1000 and 1001 members respectively. All the members have different playing strength, and the descending order of palying strengths in each club is known. Find a procedure which determines, within 11 games, who is in the 1001st place among the 2001 players in these clubs. It is assumed that a stronger player always beats a weaker one.
