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by parmenides51

1 Prove that
$$AB+PQ+QR+RP \le AP+AQ+AR+BP+BQ+BR$$
where A,B,P,Q and R are any five points in a plane.

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2 Let $n>2$ be an even number. The squares of an $n\times n$ chessboard are coloured with $\frac{1}{2}n^2$ colours in such a way that every colour is used for colouring exactly two of the squares. Prove that one can place n rooks on squares of n different colours such that no two of the rooks can take each other.

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3 For a positive integer n, $r(n)$ denote the sum of the remainders when n is divided by $1,2,...,n$ respectively. Prove that $r(k)=r(k-1)$ for infinitely many positive integers k.
