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by parmenides51

1 A cube of integral dimensions is given in space so that all four vertices of one of the faces are lattice points. Prove that the other four vertices are also lattice points.

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2 Prove that for any integer $k > 2$, there exist infinitely many positive integers n such that the least common multiple of n, $n+1$,..., $n+k-1$ is greater than the least common multiple of $n+1$,$n+2$,..., $n+k$.

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3 The set of integers is coloured in 100 colours in such a way that all the colours are used and the following is true. For any choice of intervals $[a, b]$ and $[c, d]$ of equal length and with integral endpoints, if a and c as well as b and d, respectively, have the same colour, then the whole intervals $[a, b]$ and $[c, d]$ are identically coloured in that, for any integer x, $0 \le x \le b-a$, the numbers $a+x$ and $c+x$ are of the same colour. Prove that -1982 and 1982 are of different colours

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